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Examining The Efforts of Middle School 7th Grade Students To Draw Altitude In Parallelogram And Triangle

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Abstract. The aim of this research is to examine the efforts of 7th grade students to draw altitude in parallelograms and triangles. Moreover, it is to reveal the difficulties students experience in drawing altitude. For this purpose, a case study from qualitative research methods was managed. Task-based interviews were conducted with ten 7th grade students and their efforts to draw altitude were examined. The task statuses offered to students consist of two parts: Altitudes for parallelograms tasks and altitudes for triangles tasks. The collected data were analysed with content analysis approach. As a result, it was seen that the students' understanding of altitudes were low. In addition the difficulties experienced by the students are presented under themes.

Keywords. Altitude, parallelogram, triangle.

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Geometry is one of the most important branches of mathematics. The skills gained through geometry affect us not only in mathematics, but also in all areas of our lives. For this reason, there are geometry lessons for every grade level more or less in mathematics curriculum of all countries (Duatepe and Ersoy, 2001). Geometry is an important field in mathematics education. With the perspective created by geometry, analysing and solving problems and establishing a relationship between mathematics and life can be realized more easily. Briefly, teaching geometry is as important as teaching other areas of mathematics. With this research, it is aimed to reveal the behaviours of 7th grade students regarding drawing altitudes in parallelograms and triangles. For this purpose, a research was carried out from the outcomes in the mathematics curriculum. With the results of the research, it is thought that the difficulties encountered in teaching the concept of altitude will be revealed, especially in mathematics lessons, and awareness of these difficulties will create enrichment learning environments. Knowing the results of the research by students and teachers is important in terms of learning and teaching the concept of altitude. In this study, negative information products created by students of different ideas who have learned the concept of altitude will be presented. It is stated in the literature that positive information (giving correct definitions) is not enough for learning (Baumard and Starbuck, 2005; Minsky, 1994), that knowing a subject fully can be achieved with negative information, which are the products of different thoughts (Minsky, 1994). Negative information; It is also mentioned in the literature that it is theoretically suitable for the constructivism theory (Akpınar and Akdoğan, 2010), it is valuable in learning, and it is necessary to determine the boundaries of the subject or concept to be fully learned (Heinze, 2005).

According to Turkey's Ministry of National Education Math Curriculum (MoNE, 2018); in middle school geometry topics in math lessons; in the 5th grade, basic geometry concepts, naming polygons and explaining their basic properties are taught. Calculating the rectangular area is also among the 5th grade geometry topics. In the 6th grade, students are expected to understand of angle and altitude concepts. Besides, they learn to compute the areas of parallelograms and triangles. In the 7th grade, learn the angles relations of two parallel lines cut by a transversal. Area calculations of trapezoid, rhombus and circle are also among 7th grade geometry topics. In the 8th grade, the triangle is considered deeply. The Pythagorean Theorem and its problems are also included at this grade level. In addition, geometric objects are another subject in the 8th grade (MoNE, 2018). The concept of altitude is a concept that students first saw in the 6th grade and which forms the basis for subsequent subjects (such as the area and volume of geometric objects). It is known by all mathematicians that learning a concept in mathematics lesson forms the basis of new concepts to be learned in future subjects. This study has arisen from the focus of these outcomes; "M.6.3.2.1. Creates the area relation of the triangle, solves the related problems." also that, "M.6.3.2.2. Creates the area relation of the parallelogram, solves the related problems." based on the mathematic curriculum outcomes. In this context, the problem "How are the efforts of 7th grade, middle school students in drawing the altitude in parallelogram and triangle?" is being determined as the problem of this research. With this problem, the concept of altitude will be examined in depth with a qualitative approach, and the situations are thought to directly help the teaching activities about the concept of altitude will be revealed.

When the literature is examined studies show that there are problems in learning the concept of altitude (Hershkowitz, 1987; Gutierrez and Jamie, 1999; Tomooğlu, 2017). It is stated that students try to get results without understanding concepts such as circumference, area, volume and altitude (Clements, Sarama and Battista, 1998; Stephan and Clements, 2003). Gürefe and Gültekin (2016) emphasized that the conceptual definition of altitude is not sufficiently given in school learning. It is stated that students often misunderstand the concept of altitude (Öksüz and Başışık, 2019; Senger, 2019). Gürefe and Gültekin (2016) emphasized that students' inability to detect the altitude in the triangle and the wrong constructed altitudes may cause students to have difficulties in future subjects that require the use of altitudes (area in a triangle, etc.). Hızarcı, Ada, and Elmas (2006) stated in their research that they examined the basic concepts in geometry, that students' interpretation of geometric concepts was extremely low. Researchers also emphasize that most of pre-service mathematics teachers couldn't define the altitude and couldn't draw altitude in right angle triangles and obtuse triangles. Cutugno and Spagnolo (2002) examined the misconceptions about triangle and they found that students think that altitude has to be drawn inside the triangle and divided in two parts the triangle. Gutierrez and Jamie (1999) investigated 190 students from primary teacher training school about misconception of altitude of a triangle. They determined six misconceptions: these are; altitude vs. median, altitude vs. perpendicular bisector, limitation to internal altitudes, disregard of length, fixation on side and marked base as distracter. Tomooğlu (2017) researched the topic of area measurement with seventeen 6th graders. In a part of her study results, she found that students are unsuccessful about altitude test. Öksüz and Başışık (2019) examined the misconception about polygons and quadrilaterals with 200 fifth graders. In a part of their study they found that, students could not define altitude mostly, students could draw the altitudes in prototype rectangles and parallelograms and students were quite unsuccessful on rhombus.

This study will examine the students' thoughts on drawing altitude to parallelograms and triangles with a qualitative approach and reveal which students' thoughts cause the errors about it. In this way, negative situations will be presented under themes and results that can be used directly in the teaching of the altitude concept will be obtained. This research also differs from other studies in terms of the study group (Gutierrez and Jamie, 1999; Cutugno and Spagnolo, 2002; Hızarcı, Ada and Elmas, 2006; Gürefe and Gültekin, 2016; Tomoğlu, 2017; Öksüz and Başışık, 2019; Senger, 2019). The study group of this research consists 7th grade student (age 12-13). The reason of this would be given in related part "study group". Moreover, in this study, altitude drawing situations in triangle and parallelogram are investigated. The reason for this situation is that the 7th grade students had learnt only parallelograms and triangles in line with the outcomes in the mathematics curriculum. We believe the findings of this research make an important contribution to understanding of students' behaviours for altitude concept and make a significant contribution to teaching altitude in classrooms.

Method

This section contains information about the type of research, study group, data collection process and data analysis.

Research Model

This research is a case study of qualitative research methods. Case study; it is a research method used in cases where a case is handled under normal conditions, the boundaries between the phenomenon and the content are not clear, and there are more than one evidence or data source (Yıldırım and Şimşek, 2008). Additionally, case study is where one or more situations are examined in depth with multiple data collection sources such as observations, interviews, and documents, and the themes related to the situations are defined (Creswell, 2013). Yin (2015) classifies case studies four types as 2 x 2 matrix (single-case and multiple-case studies x holistic and embedded case studies.). In this study, it is a holistic multiple-case study since students' drawing altitude in parallelogram and triangle are examined. One of the most important steps in case studies is determining the unit of analysis (Yin, 2015). The unit of analysis of this study is the thoughts in the case of determining altitude.

Study Group

The study group of the research was determined by criteria sampling, one of the purposeful sampling (also referred to as purposive sampling) methods (Creswell, 2013; Yıldırım and Şimşek, 2008). Critical situations determined by the researcher to explain the phenomenon investigated can be used for criterion sampling. However, the criteria determined for critical situations must also comply with the general harmony of the research (Creswell, 2013). The study group of the research consists of ten 7th grade students (6 Female, 4 Male). The students who selected for the research are at the school where one of the researchers works. Therefore, the researcher has detailed information about the situation of the students. This research was carried out with 7th grade students on purpose. Because, the given outcomes are learned in 6th grade in the second half of the spring term which is also the research interviews ready to apply. Therefore, it was thought that less time passed after learning the altitude concept in the 6th grade may lead to data loss. 7th grade students were selected considering those students' thoughts on the concept of altitude can be observed in a more natural environment than 6th graders. In addition, the students selected for the study group are among the 27 students, whose academic levels are higher than their classroom and who can express themselves better. That is the criteria for the sampling of study group of the research determined by the researchers. Students are studying at a state school in the Aegean region of Turkey.

Data Collection Tools

Qualitative studies require more than one data collection tool due to their nature (Yıldırım and Şimşek, 2008). The data in the study were collected through various documents. Document is any written or recorded material that provides information about the research (Merriam, 2013; Yıldırım and Şimşek, 2008). Documents of this research are audio recording, written papers and photographs. The data were collected by task-based interviews with students in the spring term of the 2018-2019 academic year. According to Koichu and Harel (2007); Task-based interview is expressed as the type of interview where the participants and the interviewer interact on a task and the task is regulated by certain norms and rules. Mathematical tasks are an important part of teacher-student communication in the teaching process (Krainer, 1993; Stein and Lane, 1996; Sullivan, Clarke, Clarke and O'Shea, 2009), task situations provide students with extensive opportunities to think, and help to understand the relationships between concepts (Stein and Smith, 1998; Antony and Walshaw, 2009). It is stated that it is important for teachers should conduct the unfamiliar task situations to their students (Polat and Dede, 2020). The task-based interview form was prepared by the researchers and consists of 16 altitude drawing tasks for 8 different parallelograms and 8 different triangles were given on chequered paper. While creating the interview form, the curriculum and literature were taken into consideration and the opinions of two experts were consulted. Getting opinions from people who have general knowledge about the research and who are specialized in related area increases reliability (Creswell, 2013). After the consultation, the special cases of parallelogram, square, rectangle and rhombus, were added to the altitude determination tasks in the interview form. In addition, considering the literature for triangle types, obtuse and right-angle triangles are also included in the tasks (Vinner and Hershkowitz, 1983; Fischbein and Nachlieli, 1998; Hızarcı et al.; 2006; Orhan, 2013).

Process

Before starting the tasks, the students were informed about the research. During the interviews of the study, the teaching activities of the school or students were not prevented. The interviews were conducted in an empty classroom in the school, when the students and the interviewer were free time. Students were interviewed one by one and all interviews were collected on the same day. The interviews were recorded with a tape recorder and the shortest interview took 10 minutes, the longest 16 minutes. Students first drew the altitudes of the parallelogram and then the altitudes of the triangle.

Data Analysis

The data of the research were analysed with the content analysis approach. In content analysis, the basic aim is to organize the collected data as the framework of similar concepts in a way that the reader can easily understand (Yıldırım and Şimşek, 2008). The validity of a research is related to the accuracy of the research results (Yıldırım and Şimşek, 2008). In this study, students' cases of determining the altitude in parallelogram and triangle were examined. This process has been recorded with a voice recorder to prevent data loss. The study group of this research consists of students at the school where one of the researchers works as a teacher. Therefore, the task-based interview form was created in a way to allow data collection, taking into account the situation of the students. In addition, while preparing this form, the opinions of two experts were consulted and the validity of the form was increased. The interviews with the students were carried out without a time limit. Therefore, sufficient time was created for data collection. In addition, the data in this study were transferred with direct quotations and photographs. According to Yıldırım and Şimşek (2008), transferring the data in detail is among the important features of validity. The reliability of a study is related to the fact that these analyses reflect the same results after repeated analysis of the data (Yıldırım and Şimşek, 2008). Due to the nature of qualitative research, each researcher's interpretation of data and perception of events may be different. The data of this study were analysed by the researchers and a teacher who is in the last year of her doctorate in mathematics education. For intercoder reliability, Miles and Huberman's (1994) intercoder reliability formula was used. For good qualitative analysis reliability, there should be 80% agreement. The data of this study were analysed by 3 coders and an 83% agreement was achieved between the analyses.

Results

This section includes the actions of the students in task-based interviews. The students in the study group were first asked whether they remember the concept of altitude or not. None of the students could give a formal definition for altitude which is "an altitude is the perpendicular segment from a vertex to the line that contains the opposite side". Four of ten students stated that they could not give the definition of altitude. The answers given by the six students are as follows:

S_1: "I remember that altitude represented by the letter h."

S_2: "It is a line close to a right angle."

S_3: "It was one of the two things used to calculate area."

S_4: "This makes the parallelogram a rectangle from its vertex."

S_5: "The one with perpendicular position."

S_6: "The right angle thing we denote by h when calculating the area."

When the student answers are examined, it can be said that the learning of the students about height is superficial. It can be said that S_4's answer has a more conceptual learning compared to other answers. In addition, it can be said that students remember the letter h in the area calculation formula when asked about the concept of altitude. It can also be said that students generally remember that the altitude is perpendicular.

Task 1: Drawing Altitude on Parallelogram

When the task papers for drawing the altitudes of parallelograms are examined, six frequently recurring codes are determined. These are;

1- Drawing the altitude correctly,
2- Drawing a diagonal instead of an altitude,

3- Thinking that the altitude must be inside of the parallelogram,
4- Inability to draw perpendicular altitude,
5- Draw any right-angle line segments instead of the altitude of the parallelogram,
6- Thinking that some parallelograms don't have altitude.

Student drawings belonging to the codes determined from the task papers are given in Figure 1, Figure 2, Figure 3, Figure 4 and Figure 5. Figure representation was not required for Code 1.

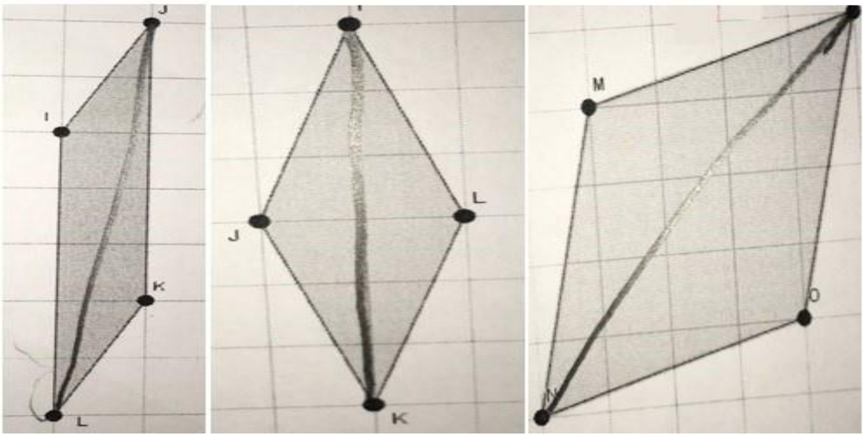

Figure 1. Drawing a Diagonal Instead of an Altitude.

When the Figure 1 is examined, it is seen that students draw diagonal instead of altitude. The frequency table for the repetition of the codes will be given in the next.

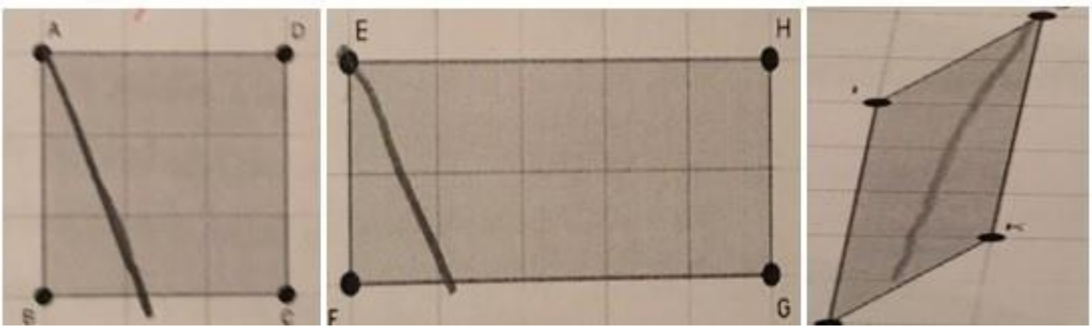

Figure 2. Thinking That the Altitude Must be inside of the Parallelogram.

When the Figure 2 is examined, it is seen that students thinking that the altitude must be inside. The reason for this situation can only be shown by drawing the altitude on the prototype parallelogram. It can be said that not drawing altitudes for other quadrilaterals, which are special forms of parallelogram, causes this misconception.

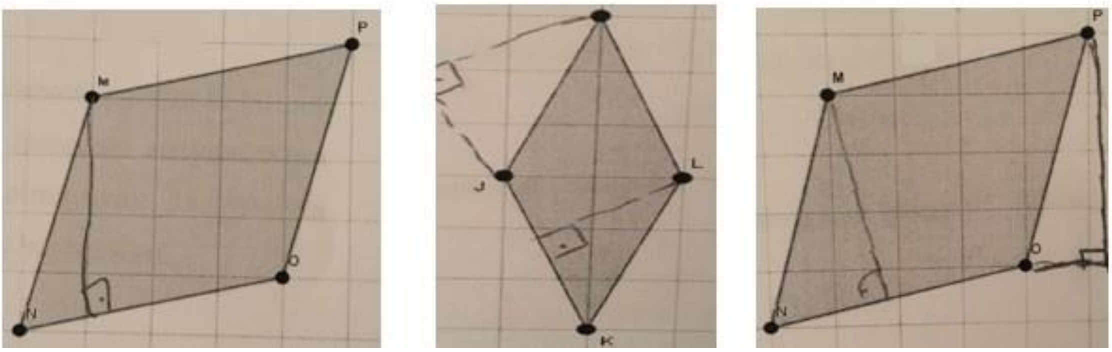

Figure 3. Inability to Draw Perpendicular Altitude.

The parallelograms in the tasks presented to the students were drawn on chequered paper as seem from figures. Therefore, students have enough data to draw a perpendicular line segment. But, it is observed that students do not use draw perpendicular line segments, which is a 5th grade mathematics subject. Also students had a ruler to use in tasks, but they didn’t.

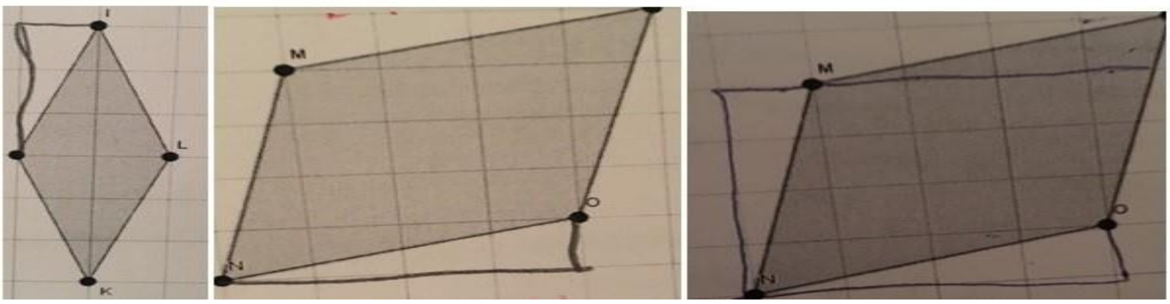

Figure 4. Draw Any Right-Angle Line Segments Instead of the Altitude of the Parallelogram.

When Figure 4 is examined, it is seen that students are stuck on the ground. They created a right-angled triangle, but did not realize that this was not an altitude. The reason for this is that the students remembered the case of cutting a part of the parallelogram and moving it to the other side to get a rectangle, but they could not use it. This points to another misconception about altitude.

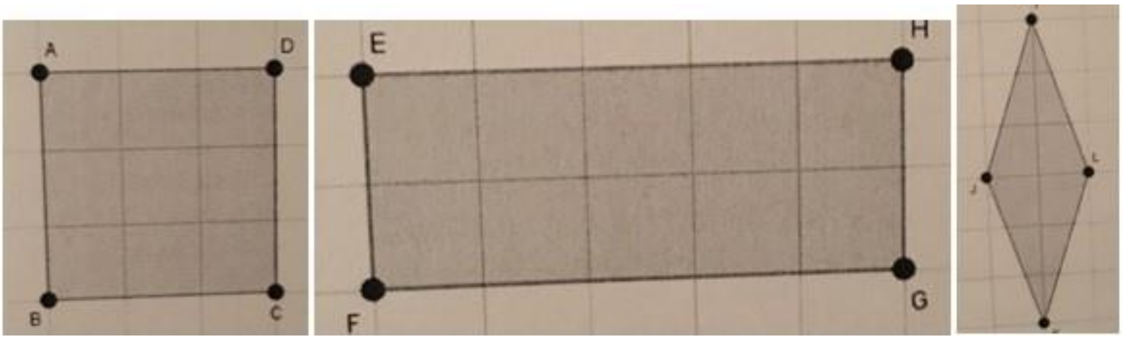

Figure 5. Thinking That Some Parallelograms Don’t Have an Altitude.

Students think that some parallelograms don’t have an altitude. They did not draw the altitudes of the parallelograms in Figure 5, which they thought did not exist. A dialogue with a student was given:

Researcher : Why didn't you draw altitude of these parallelograms?

Student : Because, they don’t have one.

Researcher : How we can calculate the area of these?

Student : This one is a square and this one is a rectangle. We can’t draw an altitude. If we would draw, it will not be perpendicular. If it would not be perpendicular, it will not be an altitude. If we can’t draw, there is no altitude.

As can be seen from the dialogue, the students’ inability to correlate square, rectangle and rhombus as parallelogram, it puts off understanding of the concept of altitude. This student knows that “altitude must be a perpendicular line” but it thinks that altitude can’t be on polygon, moreover it thinks that altitude has to be drawn from the corner. The codes frequencies of the students in the task 1 are given in Table 1.

Table 1.

Task 1 Codes Frequencies

Codes	**f**	**%**
1. Drawing the altitude correctly	49	61.25
2. Drawing a diagonal instead of an altitude	5	6.25
3. Thinking that the altitude should be inside of the parallelogram	4	5.00
4. Inability to draw perpendicular line.	9	11.25
5. Draw any right-angle line segments instead of the altitude of the parallelogram	4	5.00
6. Thinking that some parallelograms don’t have an altitude	9	11.25
Total	80	100.00

When Table 1 is examined, it is seen that the highest frequency is drawing the altitude correctly (61.25%). If this data is to be explained, it can be said that the students made the altitude drawings of the prototype parallelograms correctly, and they had difficulty in the special types of the parallelogram, such as square, rectangle and rhombus. Moreover, when the codes and quadrilateral types are examined, students drew diagonal 5 times instead of altitude (3 of them for rhombus, 1 of them for prototype parallelograms and 1 of them square); students thought that altitude should be inside (2 of them for rectangle, 1 of them for square and 1 of them for prototype parallelogram), inability to draw perpendicular (it was seen all type of parallelograms whose base was not given as horizontal or vertical), draw any right-angle line segments instead of the altitude of the parallelogram (2 of them for prototype parallelograms, 2 of them for rhombuses), thinking that some parallelograms don't have an altitude (4 of them for square, 4 of them for rectangle and 1 of them for rhombus).

Task 2: Drawing Altitude on Triangle

When the task papers for drawing the altitudes of triangles are examined, five frequently recurring codes are determined. These are;

1- Drawing the altitude correctly,
2- Thinking that the altitude must be inside of the triangle,
3- Inability to draw perpendicular altitude,
4- Dividing the obtuse angle into right angle and acute angle,
5- Thinking that some triangles don't have an altitude.

Student drawings belonging to the codes determined from the task papers are given in Figure 6, Figure 7, Figure 8 and Figure 9. Figure representation was not required for Code 1.

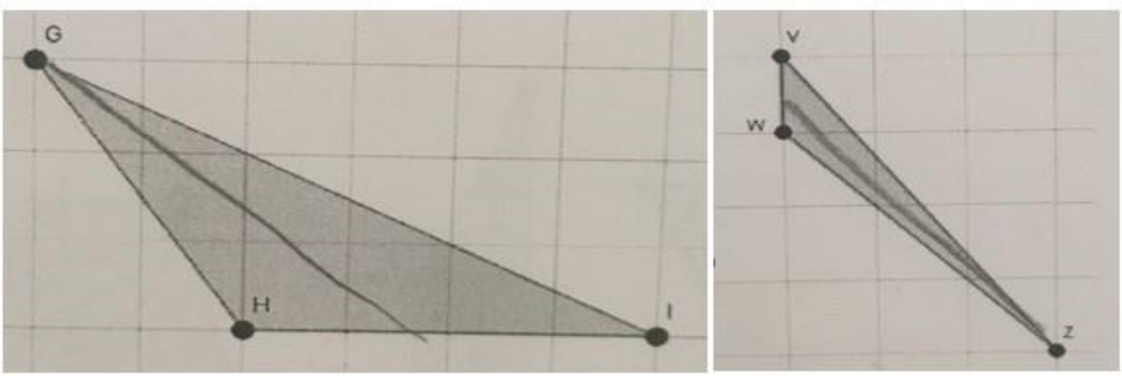

Figure 6. Thinking That the Altitude Must be inside of the Triangle.

When the Figure 6 is examined, it is seen that students thinking that the altitude must be inside of obtuse-triangle. The reason for this situation can only be shown by drawing the altitude on the acute triangle. The students tried to apply the altitude they learned in acute triangle to the obtuse triangle by over-generalizing.

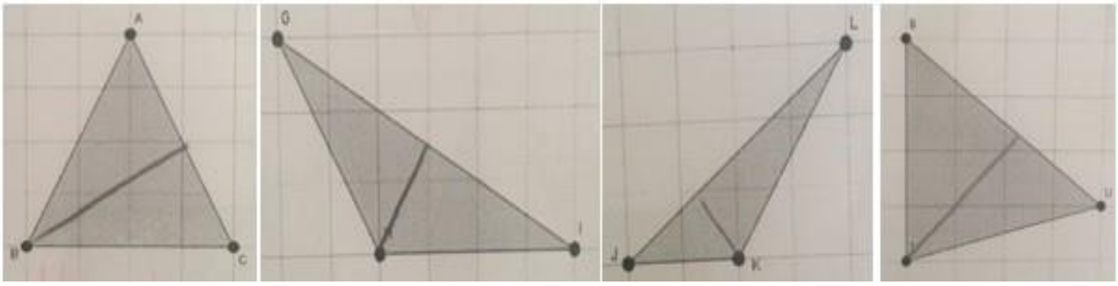

Figure 7. Inability to Draw Perpendicular Altitude.

Although the students had sufficient data to draw perpendicular line segments, they claimed that they were randomly drawing seems like perpendicular line segments as perpendicular. The students' inability to internalize the 5th grade subject of drawing perpendicular line segments prevents them from determining the altitude of the triangle.

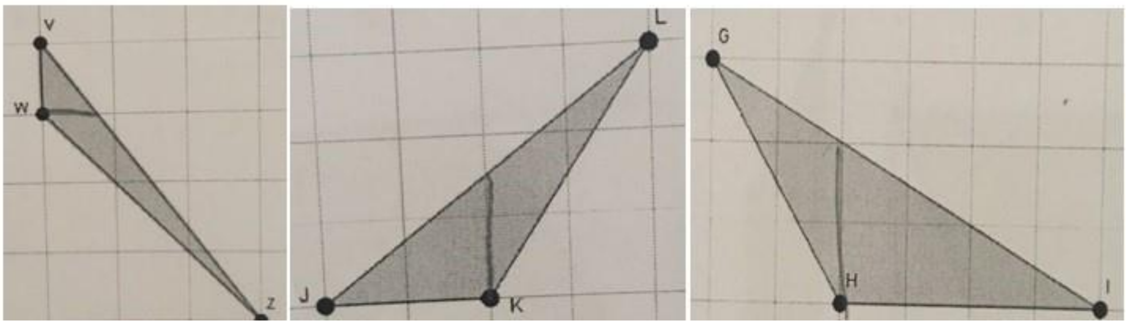

Figure 8. Dividing the Obtuse Angle into Right Angle and Acute Angle.

It was observed that the students had problems drawing altitude in triangle types, especially in obtuse triangle. When Figure 8 is examined, it is seen that students divided the obtuse angle into right angle and acute angle. Here, it is seen that the concept of altitude and 90° are associated but misplaced.

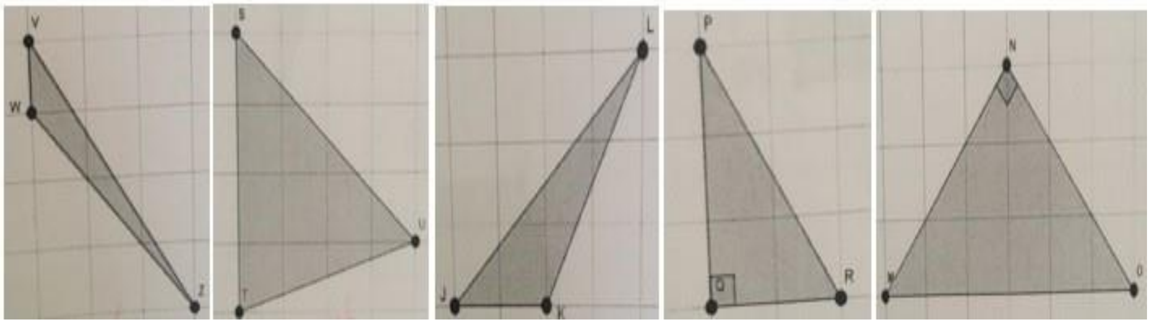

Figure 9. Thinking That Some Triangles Don't Have an Altitude.

It is seen in Figure 9 that some students think that the obtuse angle triangle and right angle triangle does not have altitude. The dialogue with a student is as follows:

Researcher : I see you left some triangles blank. Why didn't you draw altitudes on these?

Student : Because we cannot draw on these.

Researcher : Why?

Student : Because when I draw, it won't bet perpendicular. We cannot draw it because it is not perpendicular. Then we can say that there is no altitude in these triangles.

Researcher : Are you drawing from the inside of the triangle and why?

Student : If we draw from the outside of region, this line will not be the altitude.

As can be seen from the dialogue, as the students were used to determining the altitudes in acute angle triangles, they could not or had difficulty determining the altitude in right-angled triangles and obtuse angled triangles. The codes frequencies of the students in the task 2 are given in Table 2.

When Table 2 is examined, it is seen that the highest frequency belongs to the correct drawing of the altitude (47.50%). If this data is elaborated, it was found that 29 of the correct drawings belong to acute angle triangles.

Table 2.

Task 2 Codes Frequencies

Codes	f	%
1. Drawing the altitude correctly	38	47.50
2. Thinking that the altitude must be inside of the triangle	7	8.75
3. Inability to draw perpendicular altitude	14	17.50
4. Dividing the obtuse angle into right angle and acute angle	6	7.50
5. Thinking that some triangles don't have an altitude	15	18.75
Total	80	100.00

Summary, It was found that the students' knowledge of altitude was low. Therefore, It is seen that students are more successful in the ways they learn in standard form and they have difficulties in uncommon types. Six codes that repeat frequently in parallelogram tasks and five codes that repeat frequently were found in triangle tasks.

Discussion and Conclusion

In this study out aim was to examine 7th grade students' drawing altitude performances, name the repeating situations under themes and develops suggestions on this issue. This research revealed that students' had some common difficulties to draw the given parallelograms and triangles. Their efforts were coded under themes for parallelograms and triangles.

Findings of the study confirm that students had some difficulties about defining and explaining the altitude. Gürefe and Gültekin (2016) conducted a research with 33 eighth grade students, using the descriptive survey model, which is one of the qualitative research methods, in their study where they examined student knowledge on the concept of altitude. They found that most of the conceptual definitions made by the students for the concept of altitude were wrong. Moreover, Gürefe and Gültekin (2016) stated that correct definitions are at a low level of correct and wrong definitions consist of insufficient or irrelevant descriptions. It can be said that this study finding was in accord with the findings of Gürefe and Gültekin (2016). This research was carried out with seventh grade students who had learned the altitude a term ago. It can be said as a result of this study that there are some problems about learning and teaching the altitude concept, parallel to the studies Gutierrez and Jamie (1999), Hershkowitz (1987) and Öksüz and Başışık (2019).

When it comes to altitude drawing tasks for parallelograms' results it had been observed that students are successful in prototype parallelograms. In this study, altitude drawing tasks of parallelograms include rectangles, squares and rhombuses, which are special forms of parallelogram. As it stated before, naming polygons and explaining their basic properties are taught in 5th grade level. In this study, the students were 61.25% successful in drawing altitudes for parallelograms and it was observed that wrong students' drawings are mostly on non-prototype parallelograms. Gürefe and Gültekin (2016) stated that any of the participants could not draw the correct altitude for the rhombus in their study and it was also stated that students drew the diagonals in rhombus instead of the altitude. In addition to this result being parallel to our study, it was observed that the students also drew diagonals on the prototype parallelograms. Öksüz and Başışık (2019) found that students are 68% successful about drawing altitude on prototype parallelograms. It can be said that in this study, students were more unsuccessful than Öksüz and Başışık's (2019) study. Moreover, we found that students' behaviours were repeated in six codes in drawing altitude on parallelograms. These are; drawing the altitude correctly, drawing a diagonal instead of an altitude, thinking that the altitude must be inside of the parallelogram, inability to draw perpendicular altitude, draw any right-angle line segments instead of the altitude of the parallelogram, thinking that some parallelograms don't have an altitude. When learning a mathematical concept, positive information (giving correct definitions) is not enough all by itself (Baumard and Starbuck, 2005; Minsky, 2004), knowing a concept with all ways can be achieved with negative informations, which are the products of different thoughts (Minsky, 2004). In this context, we claim that it is important to know students' efforts drawing altitude on parallelograms.

When it comes to altitude drawing tasks for triangles' results it had been found that students are mostly successful in acute triangles. In other words, it can be said that students were failed in drawing altitudes in obtuse and right angle triangles. Vinner and Hershkowitz (1983) conducted a research with 189 students to understanding basic geometric concepts and they found that students were successful approximately 30% about drawing altitude for right and obtuse triangles. It is also stated in their study that students' understandings of altitude were low. It can be said that our study is showing some similar results with Vinner and Hershkowitz (1983). Similar result had been mentioned in Orhan's (2013) study. Orhan (2013) studied with 6th, 7th and 8th grade students about conceptual and procedural knowledge of area. In his study it is stated that 6th grade students' had difficulties about drawings altitude to obtuse triangle. Gutierrez and Jamie (1999) studied with 190 students from primary teacher training school about misconception of altitude of a triangle. They determined six misconceptions: these are; altitude vs. median, altitude vs. perpendicular bisector, limitation to internal altitudes, disregard of length, fixation on side and marked base as distracter. In this study, parallel to Gutierrez and Jamie's (1999) study, it is seen that the students think that the altitude must be inside of the triangle. In this study, different from the study of Gutierrez and Jamie (1999), it was observed students think that some triangles did not have altitude, students have lack of knowledge about drawing a vertical line segment on chequered paper (which is a subject that learning in 5th grade level), and dividing the obtuse angle into right angle and acute angle instead of drawing altitude codes had been determined. In this study, eight altitudes were drawn for eight different triangles in the tasks for each student. Thirty-eight of these eighty altitude drawings were found to be drawn correctly, which twenty-nine were drawn for acute angled triangles. In this respect, a parallel result had been obtained with the studies on the altitude of the triangle in the literature (Vinner and Hershkowitz, 1983; Fischbein and Nachlieli, 1998; Hızarcı et. al., 2006; Orhan, 2013)

Briefly, with this study, altitude determination situations of students' for triangle and parallelogram had been examined. As a result, it was observed that the students failed to define the altitude concept and they were more successful in drawing altitude tasks in acute-angled triangle and prototype parallelograms than in drawing altitude tasks in right, obtuse angled triangles and special forms of parallelograms (rectangle, square and rhombus). Students' repetitive wrong drawings were examined under themes and results that would deepen the teaching of the concept of altitude were shared.

Recommendations

This study was examined the question that "How are the efforts of middle school 7th grade students to draw altitude in parallelograms and triangles?" The study was carried out in suitable for the course outcomes which are "M.6.3.2.1. Creates the area relation of the triangle, solves the related problems." and "M.6.3.2.2. Creates the area relation of the parallelogram, solves the related problems.", in the mathematics lesson curriculum (MoNE, 2018) and some suggestions were developed based on the results of this study. Focusing too much on prototype drawings in teaching mathematical concepts may cause misconceptions on students. Therefore, teachers should include as many non-prototype examples as possible in their lessons. This study was carried out with ten 7th grade students and some codes were determined. If this study is repeated with more participants from different regions, this case study can turn into a "grounded theory". By developing a lesson plan that includes the codes and misconceptions seen in this study, understanding the concept of altitude can be examined with a quantitative research. In addition, it is suggested that the subject of "drawing vertical line segments on a chequered ground" should be mentioned before starting to teach the concept of altitude.

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Conflict of Interest

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Ethical Standards

We have carried out the research within the framework of the Helsinki Declaration; the participants were volunteers, informed consent has been obtained, etc. All measures have been taken regarding ethics.

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