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Investigation of Prospective Mathematics Teachers' Conceptual and Procedural Knowledge of Probability

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Abstract. The purpose of this study is to investigate prospective mathematics teachers' conceptual and procedural knowledge of probability. For this purpose survey model was used. The study group consisted of 100 prospective mathematics teachers who were studying in the third and fourth grades of Primary Mathematics Education Program of a state university. Probability Achievement Test was used to collect data related to the prospective mathematics teachers' conceptual and procedural knowledge of probability. Probability Achievement Test was consisted of two parts: Conceptual Knowledge Test and Procedural Knowledge Test. In data analysis process, an answer key and a rubric prepared by the researchers were used. As a result of the research, it was seen that the procedural knowledge test performances of the prospective teachers were relatively higher than their performance in conceptual knowledge test. However, it was observed that prospective teachers were lack of conceptual and procedural knowledge. Again, the existence of a moderate and positive relationship between the prospective teachers' conceptual and procedural knowledge test performances was revealed.

Keywords. Probability, conceptual knowledge, procedural knowledge, middle school prospective mathematics teacher.

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It is known that, today's mathematics education focuses on the ultimate goal of raising mathematically literate individuals who are equipped with the knowledge required by the century and who can use their knowledge to solve the problems they encounter in their daily life. Achievement of this goal depends on students who are doing mathematics and learning by understanding. In this context, mathematics teachers have a special importance due to the role they play in raising literate individuals. Mathematics teachers need to be able to design learning environments in which students engage in mathematics and understand what it means to do mathematics. Teachers can design such environments only if they have certain competencies.

Although there are various classifications on teacher competencies in the literature, these competencies are addressed by the Ministry of National Education (2017) under three headings as professional knowledge, professional skills and attitude and values. Here, the professional knowledge competency area includes the subject matter knowledge and pedagogical content knowledge that a teacher should have. Subject matter knowledge refers to advanced theoretical, methodological and factual knowledge. When the expression "theoretical, methodological and factual knowledge" is examined specifically in mathematics, it can be mentioned about conceptual knowledge and procedural knowledge. As a matter of fact, mathematical knowledge is addressed by most mathematics educators in two different types of knowledge as conceptual knowledge and procedural knowledge. Conceptual knowledge consists of knowledge of mathematical concepts and the relationships the individual creates between these concepts depending on his/her existing knowledge. Procedural knowledge, on the other hand, consists of two parts: one, the symbolic mathematical language, and two, the "rules, algorithms or procedures used to solve mathematical tasks" (Hiebert & Lefevre, 1986). Learning by understanding takes place as long as the conceptual and procedural knowledge are integrated with each other (Olkun & Toluk-Uçar, 2004).

One of the subjects where significant difficulties are encountered in conceptual and procedural understanding is probability. Probability is an area used when faced with various uncertainty. Therefore, knowledge of probability helps individuals to make a decision on uncertain issues. The main purpose of teaching probability is to enable learners to understand and explain probabilistic processes and thus to support the development of probabilistic reasoning, which is a basic type of mathematical reasoning.

When the literature is examined, there are many studies on subjects such as examining the effects of teaching processes based on different teaching approaches on learners' achievement and attitudes about probability (e.g. Can, 2018; Cihan, 2017; Laçin, 2014; Okuyucu, 2019; Türker, 2020), investigation of probabilistic knowledge and probabilistic reasoning levels of learners (e.g. Ata-Baran & Yenilmez, 2013; Gökkurt-Özdemir, 2017; Karaaslan & Ay, 2017; Kurt-Birel, 2017; Sarıbaş, 2019) and examining misconceptions about probability (e.g. Akbaş & Gök, 2018; İlgün, 2013; Şafak, 2016). As a result of these studies, it was revealed that understanding the subject of probability is quite difficult for both students and pre-service teachers and that learners have misconceptions about this subject. On the other hand, there are quite a few studies conducted with prospective mathematics teachers that examine their conceptual and procedural knowledge of probability. One of these studies which is conducted by Gökkurt-Özdemir (2017) focused on the most confusing probability concepts as mutually exclusive, non-mutually exclusive, dependent, and independent events. As a result of the research, it was observed that the content knowledge of most of the prospective teachers was inadequate and they confused these concepts. Similarly, in another study conducted by Kurt-Birel (2017), it was aimed to examine subject matter knowledge of prospective mathematics teachers about probability and the researcher concluded that the prospective teachers' procedural understanding about basic probability concepts was higher and that their conceptual understanding needs to be improved. Lastly, in the study conducted by Karaaslan and Ay (2017), it has been determined that prospective mathematics teachers' conceptual-procedural knowledge was not very balanced, and their conceptual knowledge was less adequate than their procedural knowledge.

Overcoming the difficulties encountered in teaching probability and developing students' probabilistic reasoning skills largely depend on teachers' subject matter knowledge of probability. Therefore, teachers need to know the procedural and conceptual aspects of probability, as well as the facilities that probabilistic reasoning will provide. In this context, the purpose of this study was to investigate prospective mathematics teachers' conceptual and procedural knowledge of probability. In line with this general purpose of the research, answers to the questions presented below were sought.

1. How is prospective mathematics teachers' conceptual knowledge of probability?
2. How is prospective mathematics teachers' procedural knowledge of probability?
3. How is the relationship between prospective mathematics teachers' conceptual and procedural knowledge, if any?

Method

Research Model

In this study survey model was used for the purpose of examining prospective mathematics teachers' conceptual and procedural knowledge of probability. A survey is a research method used for collecting data from a group to understand them well, to group them and to determine the relationships between them (Neuman, 2007).

Study Group

The study group of the study consisted of a total of 100 prospective teachers who were studying in the third and fourth grades of Primary Mathematics Education Program of a state university. The reason for working with third and fourth grade prospective teachers was that they take the Statistics and Probability course.

Data Collection

Probability Achievement Test (PAT) was used to collect data related to the prospective mathematics teachers' conceptual and procedural knowledge of probability. PAT was consisted of two parts: Conceptual Knowledge Test (CKT) and Procedural Knowledge Test (PKT). Conceptual and procedural knowledge tests were prepared in a way to measure all the objectives related to probability in the Primary Education Mathematics Teaching Program. In this context, it was aimed to include the questions in which conceptual or procedural knowledge was mainly activated during the solution process. On the other hand, in order to have more detailed information, the questions in the conceptual knowledge test were prepared as extended answer questions, while the questions in the procedural knowledge test were prepared as short answer questions. The questions prepared for the objective of "Explains an event and its probability of occurrence" are presented in Figure 1 and Figure 2 as examples of conceptual and procedural knowledge test questions.

A fair coin is tossed four times and each time the coin lands tails up. Which face is more likely to get when tossed for the fifth time?

Choose the correct answer from the options given below.

- ☐ It is more likely to get heads.
- ☐ It is more likely to get tails.
- ☐ The probability of flipping heads or tails is equal.

Explain your reasoning

....

....

Figure 1. Conceptual Knowledge Test Question.

What is the probability of rolling a prime number on a dice?

Figure 2. Procedural Knowledge Test Question.

As can be seen, in the conceptual knowledge test question, the concept of representativeness should be used mainly. In the solution process of the procedural knowledge test question probability calculation knowledge is predominantly used.

Data Analysis

During the scoring phase of the data obtained from the Probability Achievement Test, an answer key and a rubric which was prepared by the researchers were used. A rubric is considered as planning what will be scored based on which criteria. Scoring for the questions in the conceptual knowledge test is shown in Table 1.

Table 1.

Scoring of Conceptual Knowledge Test Questions

Level	Explanation	Assessment Criteria	Score
Correct Explanation	True, Complete and Clear (Very good)	Correct Answer - Generalizable Explanation	4
	Acceptable, Close to Complete, Mostly Open (Pretty good)	Correct Answer - Correct Explanation	3
Partially Correct Explanation	Incorrect, Significant Gaps, Not Too Clear (Need correction)	Correct Answer - Partially Correct Explanation	2
		Incorrect Answer - Partially Correct Explanation	1
Incorrect Explanation	Wrong, Most of them Missing, Not So Clear (The answer must be done again)	Correct Answer– Incorrect Explanation	1
		Incorrect Answer – Incorrect Explanation	0
No Explanation	No Scoring (Not ready to answer yet)	Correct Answer – No Explanation	1
		Incorrect Answer – No Explanation	0
		No Answer – No Explanation	0

While the highest score that can be obtained from the conceptual knowledge test is 152, the lowest score is 0. After scoring, the performance of the prospective teachers with a total score between 0-50 was low, the performance of the prospective teachers between 51-100 was moderate, and the performance of the prospective teachers who were between 101-152 was high. The scoring for the questions in the procedural knowledge test is shown in Table 2.

Table 2.

Scoring of Procedural Knowledge Test Questions

Level	Explanation	Assessment Criteria	Score
Correct Solution	True, Complete and Clear (Very good)	Correct Result – Correct Solution	3
		Incorrect Result– Correct Solution	2
Partially Correct Solution	Acceptable, Close to Complete, Mostly Open (Pretty good)	Correct Result – Partially Correct Solution	2
		Incorrect Result – Partially Correct Solution	1
Incorrect Solution	Incorrect, Significant Gaps, Not Too Clear (Need correction)	Correct Result – Incorrect Solution	1
		Incorrect Result – Incorrect Solution	0
No Solution	No Scoring (Not ready to answer yet)	Correct Result – No Solution	1
		Incorrect Result – No Solution	0
		No Result – No Solution	0

While the highest score that can be obtained from the procedural knowledge test is 54, the lowest score is 0. After scoring, the performance of the prospective teachers with a total score between 0-18 was low, the performance of the prospective teachers between 19-36 was moderate and the performance of the prospective teachers between 37-54 was high. In addition, the frequencies and percentage values of the total scores that the prospective teachers get from the tests were calculated.

Results

The findings of the research are described in detail below as findings related to conceptual knowledge, findings related to procedural knowledge and findings related to the relationship between conceptual and procedural knowledge.

Prospective Teachers' Conceptual Knowledge of Probability

When the prospective teachers' conceptual knowledge of probability was evaluated in general, the results obtained are presented in Table 3.

Table 3.

Conceptual Knowledge Test Results

	N	Min	Max	$\bar{X}$	s.d.
Conceptual Knowledge Test	100	17,00	86,00	55,06	14,83

According to Table 3, prospective teachers' average scores regarding CKT were determined as 55,06. This value indicated that prospective teachers were close to the lower limit of the moderate level in the context of conceptual knowledge. The distribution of prospective teachers according to the conceptual knowledge levels is shown in Table 4.

Table 4.

Distribution by Levels

Levels	Low	Moderate	High
f (%)	37	63	0

According to Table 4, it is noteworthy that there are no prospective teachers who can reach high level despite the density of prospective teachers who are at the moderate level. This situation clearly reflected the difficulty that prospective teachers had in answering the questions in CKT correctly by using their conceptual knowledge and at the same time presenting explanations about their answers. In the question related to basic probability terms, prospective teachers were expected to determine the corresponding of the terms experiment, output, sample space, event, random selection and equally likely outcomes in the problem situation. In this context, prospective teachers should use their knowledge of the mathematical meanings of basic probability terms. The score distribution for this question is presented in Table 5.

Table 5.

Score Distribution for Basic Probability Terms

	0	1	2	3	4
Experiment	7	54	1	38	0
Output	13	70	7	10	0
Sample space	14	26	3	57	0
Event	20	58	0	22	0
Random selection	34	40	0	26	0
Equally likely outcomes	8	48	43	1	0

According to Table 5, it can be said that prospective teachers were more successful in determining the corresponding of the terms of sample space and equally likely outcomes, compared to the terms of experiment, output, event and random selection. As a matter of fact when the responses of the prospective teachers were examined, it was seen that they confused the terms of event, experiment and outcome with each other and used repetitive expressions. Some of the incorrect answers to this question are presented in Figure 3.

Term	Expression corresponding to the term
Experiment	Choosing a number randomly
Output	The probability that the number is prime
Sample Space	Numbers from 1 to 13
Event	Choosing a number randomly
Random Selection	Whether the number is prime or not
Equally Likely Outcomes	?

Term	Expression corresponding to the term
Experiment	A card drawn from the bag is prime
Output	The number written on the card is prime
Sample Space	13 cards
Event	Choosing a card from the bag
Random Selection	
Equally Likely Outcomes	

Figure 3. Examples of Responses with Incorrect Explanations.

In the question related to explaining an event and its probability of occurrence, it was expected to evaluate the model in terms of its representativeness of the process of occurrence of events in a coin tossing experiment. In this context, it was necessary to use the conceptual knowledge of representativeness and interpret the probability of occurrence of the events of getting either head or tail. When the distribution of scores regarding the question was examined, it was seen that almost all of the prospective teachers (92 out of 100) marked the right option and could provide correct explanations about the reason for their preference. An example answer is presented in Figure 4.

Getting tails four times does not increase the probability of getting tails when tossed for the fifth time. The probability is the same for every coin toss.

*Figure 4***.** Example of an Answer with Correct Explanation.

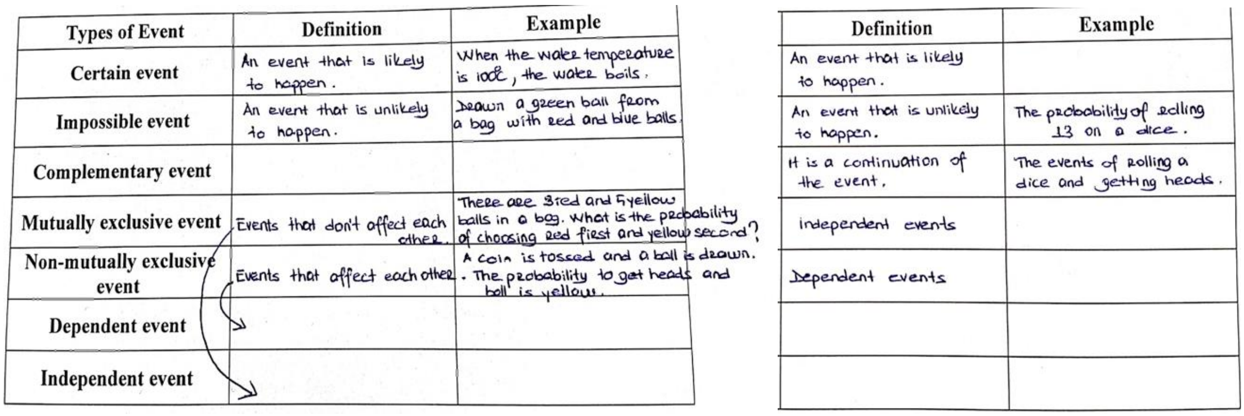
Another question reflecting the lack of conceptual knowledge of the prospective teachers was the question which was prepared to explain the types of events (certain, impossible, complementary, mutually exclusive, non- mutually exclusive, dependent, independent). In this question, the prospective teachers were expected to use their knowledge of mathematical meanings of the event types and thus explain each type of event briefly. Again, they were expected to provide a real-life example for each type of event. The score distribution for this question is presented in Table 6.

Table 6.

Score Distribution for The Question of Types of Events

	Definition					Example				
	0	**1**	**2**	**3**	**4**	**0**	**1**	**2**	**3**	**4**
Certain event	9	28	32	31	0	13	13	1	73	0
Impossible event	8	16	11	65	0	11	3	2	84	0
Complementary event	30	28	36	6	0	49	13	7	31	0
Mutually exclusive event	25	59	12	4	0	38	62	0	0	0
Non-mutually exclusive event	35	50	12	3	0	56	36	1	7	0
Dependent event	22	17	7	54	0	55	32	0	13	0
Independent event	22	23	8	47	0	57	13	0	30	0

According to Table 6, it is seen that there was no prospective teachers who get full score in the definition of event types and giving real-life examples. However, the prospective teachers were more successful in explaining the certain and impossible events and in providing a real-life example than the rest of the event types. The types of events in which prospective teachers had the greatest difficulty in explaining its mathematical meaning and presenting a real life example were mutually exclusive and non- mutually exclusive events. Again, there were prospective teachers who thought that the mutually exclusive event and the independent event were the same and similarly the non- mutually exclusive event and the dependent event were the same. This situation revealed the prospective teachers' lack of knowledge that whether the mutually exclusive/non- mutually exclusive and dependent/independent events are defined in the same sample space or not. Examples of answers with incorrect explanations are presented in Figure 5.

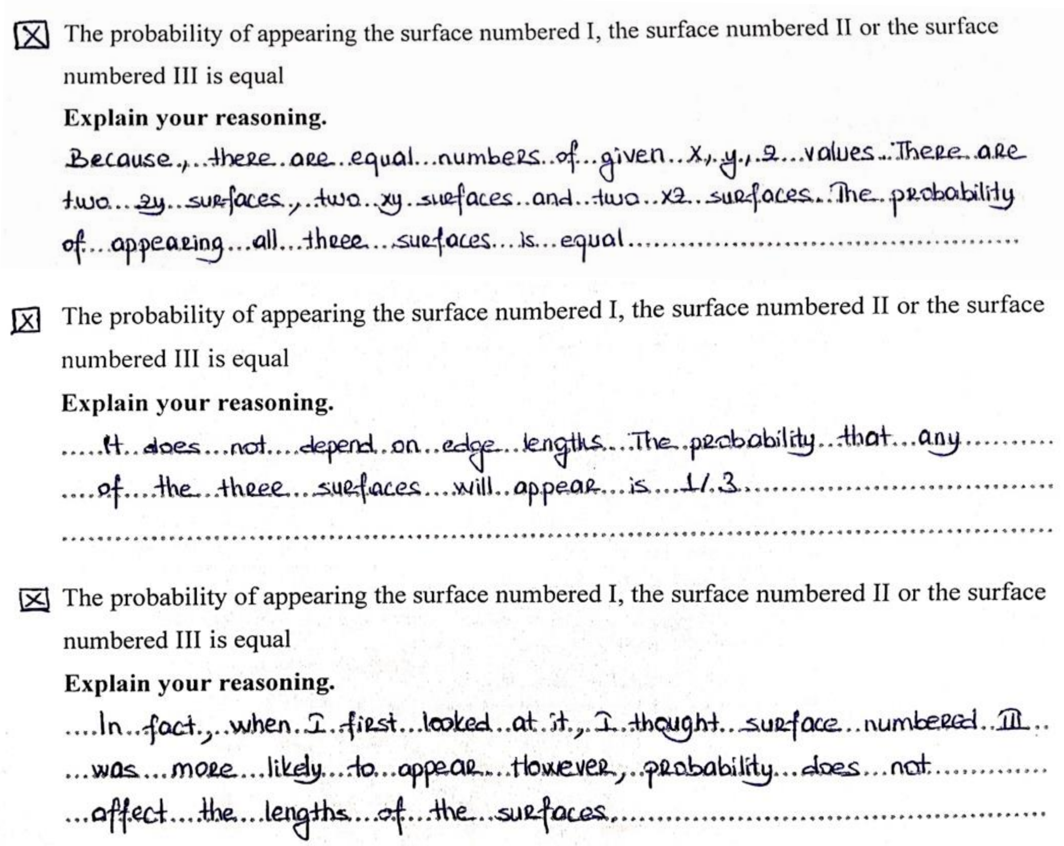


Types of Event	Definition	Example
Certain event	An event that is likely to happen.	When the water temperature is 100°C, the water boils.
Impossible event	An event that is unlikely to happen.	Drawn a green ball from a bag with red and blue balls.
Complementary event		
Mutually exclusive event	Events that don't affect each other.	There are 3 red and 5 yellow balls in a bag. What is the probability of choosing red first and yellow second?
Non-mutually exclusive event	Events that affect each other.	A coin is tossed and a ball is drawn. The probability to get heads and ball is yellow.
Dependent event		
Independent event		

Definition	Example
An event that is likely to happen.	
An event that is unlikely to happen.	The probability of rolling 13 on a dice.
It is a continuation of the event.	The events of rolling a dice and getting heads.
independent events	
Dependent events	

Figure 5. Examples of Answers with Incorrect Explanations.

In the question where prospective teachers were expected to interpret the probability of occurrence of an event using their knowledge of geometry, their lack of conceptual knowledge was once again clearly observed. This question was about the experiment of randomly throwing a box in the shape of a rectangular prism which the relationship between length, width and height of the prism was known. Here it was needed to interpret the probability of occurrence of the events of appearing either the surface numbered I, the surface numbered II or the surface numbered III. When the distribution of scores was examined, it was seen that almost half of the prospective teachers (54 out of 100) marked the wrong option and made incomplete or incorrect explanations about the reason for their preference. The answers clearly reflected the prospective teachers' lack of conceptual knowledge about the definition of probability concept. Accordingly, it was observed that they did not have the procedural knowledge of calculating the ratio of the total areas of each three different surfaces to the whole surface area. Some answers with incorrect explanations are presented as examples in Figure 6.


[X] The probability of appearing the surface numbered I, the surface numbered II or the surface numbered III is equal

Explain your reasoning.

Because, there are equal numbers of given x, y, z values. There are two zy surfaces, two xy surfaces and two xz surfaces. The probability of appearing all three surfaces is equal

[X] The probability of appearing the surface numbered I, the surface numbered II or the surface numbered III is equal

Explain your reasoning.

It does not depend on edge lengths. The probability that any of the three surfaces will appear is 1/3

[X] The probability of appearing the surface numbered I, the surface numbered II or the surface numbered III is equal

Explain your reasoning.

In fact, when I first looked at it, I thought surface numbered III was more likely to appear. However, probability does not affect the lengths of the surfaces.

Figure 6. Examples of Answers with Incorrect Explanations.

Finally, in the question about probability types (theoretical, experimental, subjective), the prospective teachers were expected to use their knowledge of mathematical meanings of the probability types and thus explain each type of probability briefly. Again, they were expected to provide a real-life example for each type of probability. The score distribution for this question is presented in Table 7.

Table 7.

Score Distribution for The Question of Types of Probability

	Definition					Example				
	0	**1**	**2**	**3**	**4**	**0**	**1**	**2**	**3**	**4**
Theoretical Probability	38	47	8	7	0	67	15	2	16	0
Experimental Probability	39	23	18	20	0	53	32	6	9	0
Subjective Probability	44	13	14	29	0	62	10	5	23	0

According to Table 7, some of the prospective teachers (average 40 prospective teachers) could not explain the mathematical meaning of probability types, while some of them made erroneous and inadequate definitions. Again, it was observed that most of the prospective teachers (on average 60 prospective teachers) had difficulty in providing a real-life example of probability types. This situation reflected their lack of conceptual knowledge about theoretical, experimental and subjective probability.

Prospective Teachers' Procedural Knowledge of Probability

When the prospective teachers' procedural knowledge of probability was evaluated in general, the results obtained are presented in Table 8.

Table 8.

Procedural Knowledge Test Results

	N	**Min**	**Max**	$\bar{X}$	**s.d.**
Procedural Knowledge Test	100	13,00	45,00	31,47	6,74

According to Table 8, prospective teachers were at around the upper limit of the moderate level. The distribution of prospective teachers according to the procedural knowledge levels is shown in Table 9.

Table 9.

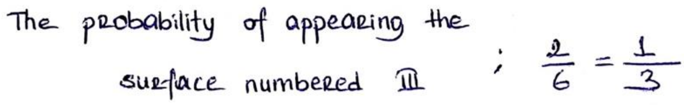
Distribution by Levels

Levels	**Low**	**Moderate**	**High**
f (%)	2	75	23

According to Table 9, although there is a density of prospective teachers who are at the moderate level, it is seen that there are also prospective teachers who can reach high level and there are very few prospective teachers who are at the low level. This situation clearly reflected that prospective teachers were relatively successful in carrying out the solution process correctly and reaching the correct mathematical result by making the necessary mathematical calculations.

In the question about explaining an event and its probability of occurrence, prospective teachers were expected to calculate the probability of getting a prime number in a dice rolling experiment. Here, when the distribution of scores was examined, it was seen that almost all of the prospective teachers (95 out of 100) could accurately calculate the probability of occurrence of the event of getting a prime number. Another question reflecting the successful performance of prospective teachers was related to calculating the probability of occurrence of a certain event and an impossible event. Most of the prospective teachers (83 out of 100), who are known to be relatively more successful in explaining the mathematical meaning of an event, were performed successfully in making mathematical calculations. Similarly, almost all of the prospective teachers (98 out of 100) reflected that they had the knowledge that the sum of the probabilities of an event and its complement is always equal to 1. On the other hand, most of the prospective teachers (76 out of 100) performed quite successfully in calculating probability of occurrence of mutually exclusive event and non-mutually exclusive event, despite the difficulty they had in explaining the mathematical meanings of them. Here, it is noteworthy that the mathematical calculations performed in most of the incorrect solutions was related to the probability of occurrence of independent events. In other words, some of the prospective teachers erroneously considered that mutually exclusive events are independent. In the question prepared for calculating the probability of occurrence of a dependent event, again, it was seen that majority of the prospective teachers (79 out of 100) could make mathematical calculations. That is they could correctly calculate the probability of the two balls drawn is blue in the experiment of drawing two balls from a bag provided that they are not replaced.

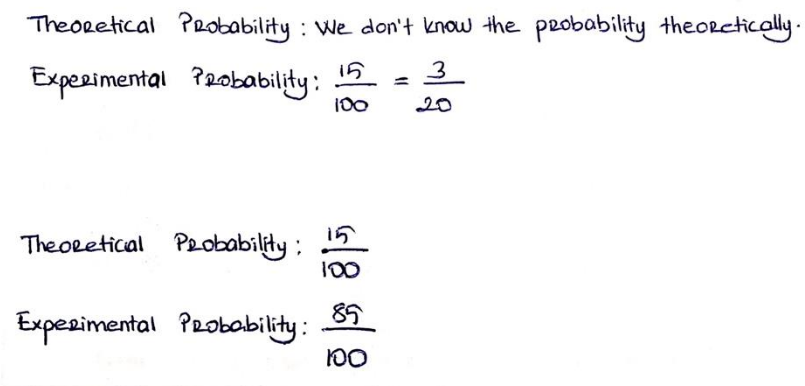
In the question where prospective teachers were expected to calculate the probability of occurrence of an event using their knowledge of geometry, it was observed that the prospective teachers had difficulties in making calculations in parallel with their lack of conceptual knowledge. Here it was expected to calculate the probability of occurrence of the event of appearing the surface numbered III in the experiment of randomly throwing a box in the shape of rectangular prism which the length, width and height of the prism was known. Here, more than half of the prospective teachers (68 out of 100) thought that the probability of appearing the surface numbered III would be found by the ratio of the number of surfaces numbered III (2) to the number of all surfaces (6). Thus, the prospective teachers clearly reflected that they did not have the procedural knowledge that the probability will be found by the ratio of the total area of surfaces numbered III to the whole surface area. An example of an incorrect answer is provided in Figure 7.



The probability of appearing the surface numbered III ; $\frac{2}{6} = \frac{1}{3}$

Figure 7. An Example of Incorrect Answer.

Finally, the question of probability types (theoretical, experimental, subjective) is about the experiment of spinning a spinner which has six equal sectors colored red, yellow, green, blue and purple. Here, it was expected to find the theoretical and experimental probability of the event of spinning red. For this the number of the sectors of any color should be taken into account. However, it was observed that more than half of the prospective teachers (69 out of 100) could not find the theoretical and experimental probability correctly. The answers of the prospective teachers revealed that they perceived the theoretical and experimental probability as complements of each other (i.e. the sum of the theoretical probability and experimental probability is equal to 1). And it was found that they ignored that the theoretical probability was calculated independently from the number of experiments performed. Some examples of incorrect answers are presented in Figure 8.



Theoretical Probability : We don't know the probability theoretically.

Experimental Probability: $\frac{15}{100} = \frac{3}{20}$

Theoretical Probability : $\frac{15}{100}$

Experimental Probability: $\frac{85}{100}$

Figure 8. Examples with Incorrect Answers.

The Relationship Between Prospective Teachers' Conceptual and Procedural Knowledge of Probability

Whether there is a relationship between prospective teachers' conceptual and procedural knowledge of probability was examined with the Pearson Product Moment Correlation Analysis and the results are presented in Table 10.

Table 10.

The Relationship Between Conceptual and Procedural Knowledge of Probability

	N	r	p
Conceptual Knowledge-Procedural Knowledge	100	0.358	< 0.001

A correlation coefficient between 0.00-0.30 is defined as "low correlation", between 0.30-0.70 "moderate correlation" and 0.70-1.00 as "high correlation" (Büyüköztürk, 2010). Accordingly, it can be said that (although very close to the lower limit) there is a moderate and positive relationship between the conceptual and procedural knowledge of the prospective teachers.

Discussion and Conclusion

Considering the importance of the development of probabilistic reasoning and the role of mathematics teachers' subject matter knowledge in supporting students' probabilistic reasoning, in this study it was aimed to examine prospective mathematics teachers' conceptual and procedural knowledge of probability. Within the scope of this study the performance of 100 prospective teachers was examined by using conceptual and procedural knowledge test questions about probability. Thus, it has been studied to determine how the prospective teachers use their conceptual and procedural knowledge and the difficulties they had in problem solving process. In this context, it has been revealed that prospective teachers were lack of conceptual knowledge about the mathematical meanings of basic probability terms, types of event and types of probability. For instance, it was observed that most of the prospective teachers could not provide mathematically meaningful explanations about mutually exclusive-non-mutually exclusive and dependent-independent events. Similarly, in the question item which required explaining the mathematical meanings of types of probability and presenting a daily life example, the low performance of the prospective teachers was noteworthy. In this context, the low achievement observed in conceptual knowledge test questions showed that prospective teachers underperformed. On the other hand, it has been observed that prospective teachers were relatively more successful in solving the procedural knowledge test questions. That is, prospective teachers were generally able to do necessary mathematical calculations for the probability of an event of occurrence. For instance, unlike the difficulties they had in calculating the probability of occurrence of mutually exclusive and non-mutually exclusive events and explaining the mathematical meanings of these events, the prospective teachers were able to do calculations. These results obtained in this study regarding prospective teachers' conceptual and procedural knowledge of probability is in line with the research results that pre-service teachers' lack of conceptual knowledge related to probability and that their procedural knowledge is relatively good

(Karaaslan & Ay, 2017; Kurt-Birel, 2017; Gökkurt-Özdemir, 2017). As is known, prospective teachers will be responsible for supporting middle school students' probabilistic reasoning and they are expected to make their teaching in this direction. However, the prospective teachers failed to show the expected performance in solving conceptual and procedural knowledge test questions due to the lack of knowledge they are responsible for developing in students. Therefore, this study calls into question the performance of prospective teachers in raising individuals with advanced probabilistic reasoning skills. Here, a suggestion of conducting studies to support pre-service teachers' conceptual knowledge of probability can be offered. For instance, learning environments in which real life situations are addressed can be designed for learners. Similar studies can be carried out with pre-service teachers who take the Probability and Statistics Teaching course in the primary mathematics education undergraduate program, and both subject matter knowledge and pedagogical content knowledge can be examined in detail.

On the other hand, the correlation coefficient was calculated as 0.358 as a result of the quantitative analysis carried out in order to examine the relationship between the conceptual and procedural knowledge test performances of prospective teachers. Although this value is very close to the low correlation score range (0.00-0.30), it indicates the existence of a moderate and positive relationship. In this way, the need arises for studies that examine the relationship between pre-service teachers' conceptual and procedural knowledge of probability qualitatively.

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Conflict of Interest

It has been reported by the authors that there is no conflict of interest.

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Ethical Standards

We have carried out the research within the framework of the Helsinki Declaration. The consent forms were utilized. The participants were informed about the study and volunteered to participate.

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