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The Relationship between 9th Grade Students' Symbol Sense Behaviors, Algebraic Thinking Skills and Academic Achievement: A Case Study

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Abstract. The aim of this study is to investigate the relationship between ninth grade students' symbol sense behaviors, algebraic thinking skills and academic achievement. To achieve this aim, a qualitative research approach known as case study design was employed. A total of three students studying in a high school in Gümüşhane province constituted the study group of the research. In the study, considering the opinions of the mathematics teacher conducting the course and the academic achievement levels of the students in the mathematics course, one student from each achievement level was selected as low, medium and high academic achievement level. The data were acquired from five research inquiries in the literature and adapted in line accordance with expert perspectives. The data were analyzed using thematic coding with an analysis table prepared in line with expert opinions. Students with high achievement level showed symbol sense behaviors at the desired level by using symbols in a flexible and fluent way, while students with low and medium achievement level could not exhibit symbol sense behaviors at the desired level. Students with high academic achievement have advanced algebraic thinking skills. The algebraic thinking skill behaviors of students with low academic achievement are more in the form of rote stereotyped signs. As a result, students' algebraic thinking skills and symbol sense behaviors were found to be compatible with their academic achievement levels. In addition, it was concluded that there was a positive relationship between algebraic thinking skills and symbol sense behaviors.

Keywords. Symbol sense behaviors, algebraic thinking skills, academic achievement, 9th grade students.

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Since algebra is closely related to the construction, development, and communication of knowledge in all domains of mathematics (NCTM, 2000), comprehension of algebra holds significant importance within the realm of mathematical education (Chow, 2011). Algebra, being a potent instrument, exerts influence across various branches of mathematics and holds a pivotal role in the educational process of mathematics across different proficiency levels (Lacampagne, 1995; Kieran and Drijvers, 2006; Irwin and Britt, 2007). In this context, algebra has been defined as a gateway or gate keeper to advanced mathematics education for students from past to present, but this door is closed for many students (Lacampagne, 1995; Kaput, 1999; Toluk-Uçar, 2018). To be successful in algebra, algebraic thinking skills need to be developed (Kieran, 2004).

Algebraic thinking, which starts to develop from primary school, is one of the most vital and main elements of mathematical thinking and reasoning (Toluk-Uçar, 2018). In the literature, algebraic thinking process is known as the process of transition from real and mathematical contexts to structure. The evolution of the human ability to comprehend and use symbols is part of this process. The foundation of algebraic thinking begins in elementary and middle school with mathematical recognition of number patterns and progresses towards generalization. Effective algebraic thinking necessitates proficient symbolization and generalization abilities (Sibgatullin, Korzhuev, Khairullina, Sadykova, Baturina and Chauzova, 2022).

According to Sibgatullin et al., (2022), algebraic thinking typically encompasses the act of generalizing arithmetic procedures and gains intricacy with advancement, emphasizing variables. The five fundamental elements of algebraic cognition (*a. Generalization and formulation of arithmetic operations, b. Analyzing and converting specific equality issues using inverse operations and fundamental syntax, c. Examination of mathematical structures, d. Exploration of relations and functions that include both numbers and variables and e. Algebraic language and notation*) are discussed in various research studies (Usiskin, 1999; NCTM, 2000; Radford, 2000; Schliemann, Carraher and Brizuela, 2006; Stephens, Blanton, Knuth, Isler-Baykal and Gardiner, 2015).

According to Kamol (2005), the fundamental elements of algebraic thinking encompass three core skills: Notation, model (pattern), and variable. Notation entails the capacity to employ tables, graphs, symbols, etc., within a specified problem. The model comprises skills in pattern formulation and generalization. The concept of variable involves comprehending the function of variables in generalized numerical contexts. Kriegler (2007), categorizes algebraic thinking into two primary elements: the formation of mathematical thinking instruments and the examination of fundamental algebraic concepts. The mathematical thinking instruments encompass skills for problem-solving, abilities for representation, and cognitive patterns for analysis, all of which pertain to quantitative reasoning. Fundamental algebraic concepts delineate the subject area in which the instruments for mathematical thinking are cultivated and encompass the subcategories algebra as a generalized form of arithmetic, algebra as a mode of expression, functions, and algebra as a tool for mathematical modeling.

When the basic components of algebraic thinking are examined, it is seen that algebraic thinking requires mathematical reasoning within a mental framework (Sibgatullin et. al., 2022). According to Piaget, students between the ages of 7 and 15 are in the formal processing stage. These students have serious difficulties in forming basic mathematical ideas. Among these difficulties is the concept of variable, which is one of the basic algebraic ideas. It is clear that students' difficulties in understanding and using symbols effectively will negatively affect the development of algebraic thinking and reasoning. Symbol sense, facilitates the process of algebraic reasoning (Somasundram, Akmar and Eu, 2019). Symbol sense refers to students' understanding and manipulation of algebraic symbols, which is crucial for successful algebra learning (Naidoo, 2009). In this context, teachers need to know how students use and interpret symbols in algebraic thinking. From this point of view, it is also important for students to have a developed symbol sense. What is symbol sense?

Symbol Sense

Symbol sense is a recent term in maths education, similar to number sense (Rycroft-Smith and Macey, 2022). In light of the research conducted on "number sense" during the 1980s and 1990s, the notion of expanding number sense beyond elementary arithmetic to encompass algebraic concepts in educational settings serves as a foundational element for the development of symbol sense (Arcavi, 1994). Today, just as the main focus of learning and teaching arithmetic is not the correct performance of operations on numbers, the main focus of learning and teaching algebra is not the correct symbolic manipulations. Students with a developed symbol sense can flexibly read and interpret letter symbolic expressions and fluently use the complex symbolic language of mathematics. Similar to the concept of number sense, symbol sense entails a deep understanding and perceptiveness when manipulating symbolic representations and mathematical operations. Nevertheless, the educational resources and pedagogical strategies aimed at cultivating symbol sense lack the same level of detail and elaboration in comparison to those dedicated to promoting number sense (Fey, 1990). Arcavi (1994) contended that the evolution of symbol sense is shaped by its interplay with various faculties, including numerical perception, operational understanding, and visual interpretation, ultimately resulting in progression and transformation. Symbol sense behaviors and their explanations, have main components. These main components named as "friendliness with symbols", "reading and using symbolic expressions", "designing symbolic expressions", "symbol selection", "checking symbol meanings" and "symbol context" (Arcavi, 1994; 2005). The descriptions of these components are as follows: a) *Friendliness with symbols:* knowing when to intuit when a symbol is needed in the process of solving a problem, and vice versa, knowing when to abandon a symbol, b) *Reading and using symbolic expressions:* Pre-examination of symbols with the expectation of sensing problems and problem meanings and checking the contrast between meaning-making and symbolic use, c) *Designing symbolic expressions*: Conveying the verbal or visual data essential for advancing in a task and constructing symbolic expressions, d) *Symbol selection:* The act of selecting a symbol representation from various options for a given problem, e) *Checking symbol meanings:* This task entails the examination of the interpretations of symbols either prior to or during the implementation of a process, the resolution of an issue, or the analysis of findings, and juxtaposing the symbolic interpretation with one's own intuitive understanding of the anticipated result and f) *Symbol Context*: The recognition that symbols possess varying functions in diverse contexts, such as variables, is crucial. The utilization of a particular variable may necessitate distinct interpretations across various problem scenarios.

Symbol sense is the understanding of situations in which symbols can be used. Developing symbol sense in algebra is an important way to improve students' abstraction and generalisation skills (Arcavi, 1994; 2005; Bokhove and Drijvers, 2010; Jupri and Sispiyati, 2020). In this respect, it is expected that symbol sense is given the necessary importance and reflected in the applications. The findings from the study carried out by Dede and Argün (2003) demonstrated a recurrence in the challenges faced by both local and global students when it comes to comprehending algebra. Students' difficulties in learning algebra come from at least two sources. First and foremost, the acquisition of algebraic knowledge necessitates students to grasp the intricate system of mathematical symbols, a system that starkly differs from their prior cognitive encounters. Therefore, students need to understand symbols when learning algebra. Second, algebra is a subject that requires students to develop abstract reasoning and problem solving (Kusaeri, 2012). The concept of symbol sense pertains to the comprehension of symbols within the realm of algebra, as discussed by Bokhove (2010). In order for students to learn algebra, symbol sense should be at the heart of algebra and algebra teaching should be oriented towards it (Arcavi, 1994).

Despite the presence of overlapping yet conflicting perspectives regarding the conceptualization of algebra and algebraic reasoning, there is consensus among scholars that proficiency in algebraic thinking necessitates adept symbolization and generalization abilities (Kaput, 2008). Based on the findings by Mason (2008), it is recommended that educators guide young learners as they embark on their journey of numerical exploration, ensuring that the process of deriving meaning from numbers gradually transitions towards the realm of algebraic reasoning. Algebraic thinking is cultivated as a result of the identification of numerical patterns in arithmetic, leading to the child's ability to make generalizations. Over the course of time and through focused educational interventions, the algebraic cognition of juvenile learners progresses in sophistication. When it comes to recognizing patterns and formulating mathematical abstractions, it is imperative for educators to tactfully steer students towards adopting an algebraic mindset. Teachers must possess a thorough understanding of their students' proficiency in algebraic thinking and their approach towards mathematical problem-solving. This knowledge is crucial for comprehending the cognitive development and logical reasoning of students, thus facilitating their engagement in substantial mathematical tasks at an advanced academic stage (Sibgatullin et al., 2022). Students' symbol sense entails their ability to recognize and interpret mathematical symbols (Mutammam and Wulandari, 2023). Understanding mathematical problems relies heavily on grasping the significance of symbols in a given context (Wardah, Utomo and Putri, 2021).

The ninth grade represents an important step in the transition from middle school to high school. As recommended by the National Research Council-[NRC] (1989), it is emphasised that the main purpose of mathematics education in the transition from middle school to high school is to develop symbol sense and to further strengthen number sense (Keller, 1993). In this context, this study was conducted with ninth grade students.

When the literature is examined, it is evident that symbol sense is a crucial ability in effectively managing symbolic expressions and algebraic operations. Furthermore, it significantly contributes to the resolution of algebraic problems (Kop, Janssen, Drijvers and van Driel, 2020; Rini, Hussen, Hidayati and Muttaqien, 2021); it is seen that it affects students' general mathematical competences (Sugilar, Kariadinata and Sobarningsih, 2019) as it is the ability to understand and use mathematical symbols effectively (Rini et al., 2021), which is very important for solving algebraic problems.

According to Arcavi (1994), symbol feeling is a feeling that occurs at all stages of problem solving. One of the most widely used theories in the problem-solving process is Polya's problem solving theory. Polya (1945) proposed that the process of problem solving can be broken down into four key stages: understanding the problem, making a plan, implementation and rechecking. This study is believed to enhance the analysis of students' algebraic skills and cognitive stages across various levels of achievement when tackling problems through the lens of symbol sense.

From this perspective, the aim of the study is to investigate algebraic thinking skills of ninth grade students at different achievement levels (low, medium and high) through out of the symbol sense behaviors at Polya (1945)'s problem solving model. It was acknowledged that the expert evaluations regarding the challenges encountered in the methodologies employed in the research were both appropriate and adequate. It was presumed that the students provided objective and accurate responses to the inquiries posed to them throughout the interview process. A notable limitation of the study was that it was restricted to 9^{th} grade students and five specific challenges. Additionally, another limitation of the study was that the stages of problem-solving exhibited by the students were analyzed solely through the lens of the Polya model, with a focus exclusively on the symbolic reasoning behaviors demonstrated by the students during these problem-solving phases. The sub-problem formed in accordance with the purpose and problem statement of the research can be given as follows:

- What are the algebraic thinking skills of ninth grade students at different academic achievement levels (low, medium and high) thorough out of the symbol sense behaviors at problem-solving?

Method

This study investigates the relationship between symbol sense behaviors and algebraic thinking skills in terms of academic achievement. It is structured as a case study. A case study pertains to an endeavor to examine a present occurrence, an extensive depiction, and evaluation of a restricted system, individual, group, or a specific event or situation in great detail by centering on individuals and groups within the realm of authentic situations, particularly when the distinctions between the occurrence and its context are ambiguous (Yin, 1994). This case study examines each activity separately for each student while considering the situation as a whole.

This study; is related to the process since it aims to investigate students' symbol sense behaviors and algebraic thinking skills. The content is characterized by its descriptive nature as it elucidates the cognitive processes and behavioral actions undertaken by students participating in the activities extensively. It is inductive in the sense that it deals with students' symbol sense behaviors based on algebraic thinking processes. One of the researchers acted as both teacher and researcher during the activity.

In this research, a case study was determined as the most appropriate method as it aims to examine in depth the relationship between students' algebraic thinking skills from the perspective of academic success through out of the symbol sense behaviors at problem solving.

Study Group

The investigation was carried out involving three students enrolled in the 9^{th} grade at a public high school in Gümüşhane province. The school where the study was conducted is a school with a medium socio-economic level and is located in a rural area. In order to conduct the study in the designated state school, the necessary permissions were obtained from the students and their families from the Gümüşhane Provincial Directorate of National Education. In addition, permission was obtained from the Social and Human Sciences Scientific Research and Publication Ethics Board for the study. The utilization of the "maximum diversity" sampling technique, a purposeful sampling approach, was employed to identify the research participants, which comprised three students. The primary objective is to establish a compact working group and to effectively represent the diverse student population relevant to the research topic. The objective is not to ensure diversity for the purpose of making generalizations; instead, it is to investigate whether there exist common or shared phenomena and distinctions among various scenarios, unveiling diverse aspects of the issue based on diversity (Yıldırım and Şimşek, 2016). Nevertheless, the maximum variation approach strives to uncover and recognize the primary themes that encompass a multitude of distinctions linked to the occurrence or concept being examined (Neuman, 2014). The study group, which was determined by using the purposeful sampling method, consists of three 9^{th} grade students. The reason for choosing this group is that the selected students are a heterogeneous group in terms of academic success and, according to the teacher, they are students who are thought to have good communication skills and can express themselves. In order to determine the algebraic thinking levels of the students, the “algebraic thinking level determination test” (Altun, 2005) was used with permission and after the test was applied, the algebraic thinking levels of the students were determined. For this purpose, one student from each of the three levels (low, medium and high) was selected for the study group according to, *a) the opinions of the mathematics teacher conducting the course, b) The extent of algebraic reasoning proficiency following the administration of the algebraic reasoning assessment. c) the academic success of the students in the mathematics course were taken into consideration in the selection of the students.* Since the acquisitions related to algebra learning domain in the mathematics curriculum implemented in our country were first addressed in the 6^{th} grade, the student's academic achievement in the mathematics course was determined by the 6^{th}, 7^{th} and 8^{th}

grade grades. In terms of research ethics, the study group was formed on the basis of volunteerism and student and parent permissions were obtained.

In the presentation of the data, the code names Serkan, Eda and Yıldız were used to characterise the students with low, medium and high achievement levels, respectively, instead of their real names. In this study; Serkan coded as (LALS-Low Achievement Level Student), Eda coded as (MALS-Medium Achievement Level Student) and Yıldız coded as (HALS-High Achievement Level Student).

Data Collection Tools

In the first pilot application, a total of eight symbol sense behaviors were determined by obtaining the opinions of expert mathematics educators in the field. Then, for each of these behaviors, a total of fourteen problems were created by adapting the problems related to the relevant behavior in the literature. The data collection tool consisting of these problems was applied to six 9^{th} grade students on a voluntary basis. As a result of the application, the researcher observed that four of the problems in the study could be solved by very few students, that the study took a long time and tired the students. In addition, the researcher and the experts agreed that it was not appropriate to create problems for each symbol sense behavior and that it would be more appropriate to examine symbol sense behaviors in the created problems. In addition, the experts stated that applying semi-structured interview forms in the following pilot application, including pre-interview questions that included the students' opinions about mathematics course, algebra, mathematical problems and mathematical symbols before solving the problems; and a final interview form that included their opinions about the problems applied and the symbols encountered after solving the problems would be effective in terms of enriching the study. The study, which included pre-interview and post-interview questions prepared in line with expert opinions and eight problems, was conducted with 3 randomly selected students from the 9^{th} grade. The application lasted a total of 1 week, and lasted an average of 30 minutes for each student. Since the symbol sense behaviors exhibited in three problems in the application overlapped with the symbol sense behaviors exhibited in the other five problems, it was decided to apply five problems in the final version of the study after consulting the experts.

Final data collection tool of the research was collected with a problem form consisting of; a) six pre-interview questions, b) five problems in the literature and adapted in line with the expert opinions and c) three post-interview questions.

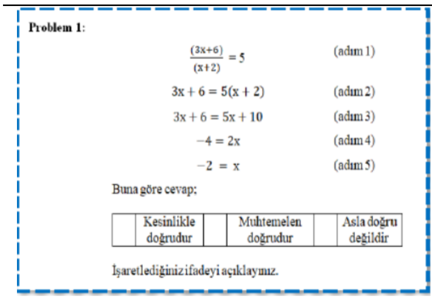
The study involved the administration of six pre-interview inquiries (*i. are you fond of mathematics? Could you provide a rationale for your preference?, ii. in what academic year do you* *believe you excel the most in mathematics? Kindly elaborate on your reasoning., iii. Is there a particular branch of mathematics that captivates you (e.g., numbers, algebra, geometry, measurement, counting, and probability)? If so, please elucidate on the reasons behind your choice., iv) Do you encounter challenges when tackling mathematical problems?, v) How would you define the term 'algebra' in the realm of mathematics?, vi) When contemplating the concept of a 'symbol' in mathematics, what comes to mind? Which symbol or symbols do you frequently employ?*) aimed at eliciting the students' opinions regarding the mathematics curriculum, their stances on problem-solving, and their attitudes towards mathematical symbols, all of which were shaped by their previous encounters.

The five study problems in the study were sometimes given as a real-life problem and sometimes as a situation requiring algebraic expression. The original researchers (Arcavi, 1994; Kenney, 2008) named these tasks as problems. For this reason, by adhering to the citations in the literature, it was preferred to use the term problem for each task instead of expressions such as question or item. Baki (2008) defined algebra as 'making generalisations', 'using operations and algorithms to solve problems', 'studying relationships between quantities' and 'studying abstract structures such as groups, rings and vector spaces'. Each component in this definition is described as school algebra understanding in Usiskin's (1999) study. The study problems used in this study, are based on the algebraic components of Usiskin's (1999) and Baki's (2008) studies as: “A) Generalisation, B) Using Operations and Algorithms to Solve the Problems, C) Quantitative Relationship”.

The five study problems used in this study were named as Problem 1, Problem 2, Problem 3, Problem 4 and Problem 5, in Table 1 with two columns as characteristics of problems and algebraic components.

Table 1.

Characteristics of Problems

Study Problems	**Characteristics of Problems**	**Algebraic Components**
 **Problem 1:** $\frac{(3x+6)}{(x+2)} = 5$ (adım 1) $3x + 6 = 5(x + 2)$ (adım 2) $3x + 6 = 5x + 10$ (adım 3) $-4 = 2x$ (adım 4) $-2 = x$ (adım 5) Buna göre cevap; [ ] Kesinlikle doğrudur [ ] Muhtemelen doğrudur [ ] Asla doğru değildir İşaretlediğiniz ifadeyi açıklayınız.	This problem is a rational equation adapted from Arcavi's (1994) study.	In the solution process of this problem students are expected to “*use operations and algorithms to solve the problem*”.

Problem 2: $\|3x - \frac{2}{4}\| + 1{,}2 > 5$ eşitsizliğinde x kaçtır?	This problem is a linear inequality problem adapted from Kenney's (2008) study.	In the solution process of this problem students are expected to *"use operations and algorithms to solve the problem"*.
Problem 3: **Bir üniversitede, profesörlerin 6 katı kadar öğrenci vardır.** "P" profesör sayısını, "Ö" öğrenci sayısını göstermek üzere bu durumu temsil eden denklem/denklemler aşağıdakilerden hangisi ya da hangileridir? No — Denklem 1 — P–6–Ö 2 — 6Ö=P 3 — 6P–Ö 4 — Ö=6=P 5 — $\frac{Ö}{6}$=P 6 — $\frac{P}{6}$=Ö	This problem is an algebraic verbal problem which was known as the "students and professors" problem in the literature (Rosnick and Clement, 1980; Clement, Lochhead, and Monk, 1981; Clement, 1982; Arcavi, 1994).	In the solution process of this problem students are expected to see "*quantitative relationship".*
Problem 4: Ardışık üç sayının toplamı 54 ise küçük sayı kaçtır?	This problem is a verbal problem adapted from Arcavi's (1994) study.	In the solution process of this problem, students are expected to use the *"generalisations"* "$n-1, n, n+1$" *or* "$n, n+1, n+2$" of three consecutive natural numbers in accordance with the problem context.
Problem 5: Bir dikdörtgenin **bir kenarı** % 10 arttırıldığında ve **diğer kenarı** % 10 azaltıldığında dikdörtgenin alanında nasıl bir değişim meydana gelirdi? Aşağıda verilen seçeneklerden sizin için uygun olanı işaretleyiniz ve cevabınızı açıklayınız. A) Bir değişim olmazdı, çünkü... **AÇIKLAMA:** B) Alan artar, çünkü... **AÇIKLAMA:** C) Alan azalır, çünkü... **AÇIKLAMA:**	This problem is a verbal problem adapted from Arcavi's (1994) study.	In the solution process of this problem, students are expected to see *the "quantitative relationship".*

At the end of the study, a series of three semi-structured post-interview questionnaire (*i. what types of symbols such as variables, coefficients, constants, or algebraic operations did you come across in the problems explored in the study?, ii. which particular problem presented as more manageable for you to resolve? What were the underlying reasons for this ease?, iii. which specific problem posed the greatest challenge for you in terms of finding a solution?*) were asked to the students. These interviews were conducted to gain a comprehensive understanding of the students' approaches to problem solving and to delve into the cognitive progression of individuals when confronted with algebraic challenges.

Process

In qualitative research, the researcher uses an inductive data collection process and makes the data meaningful by grouping them into codes, then themes and finally broader perspectives (Creswell, 2017). The data collection process of this research consists of six pre-interview questions, five problem questions and three post-interview questions. The data collection process in the problem research is given in Table 2.

Table 2.

Interview Durations during Data Collection Process

Data Collection Tools	Yıldız (HALS)	Eda (MALS)	Serkan (LALS)
Pre-interview inquiries	9’	5’	2’
Problem 1	6’	7’	4’
Problem 2	4’	5’	5’
Problem 3	2’ 15’’	3’	2’ 10’’
Problem 4	1’ 25’’	2’ 25’’	1’ 25’’
Problem 5	3’	5’	4’ 7’’
Post-interview inquiries	3’	2’	2’ 40’’
Total Duration	28’ 40’’	29’ 25’’	21’ 22’’

Table 2 presents the interview durations for the six pre-interview questions, five problem questions and three post-interview questions conducted with each participant during data collection process.

Data Analysis

As this is a case study, the analysis of the data followed the general steps of qualitative research, which include organizing, coding, summarizing, and interpreting the collected documents and field notes (Merriam and Tisdell, 2016; Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz and Demirel, 2017). In order to ensure the validity of this qualitative study, the researcher spent enough time in the environment where the study was conducted and had long-term interactions with the study group. First, after the data was collected, each participant was interviewed twice to check the accuracy and integrity of the results, thus trying to support the accuracy and consistency of the findings. In addition, in this study, the questions used in the semi-structured interview forms, the selection and implementation of the data collection tools, and the examination of the applications by experts in the field of education were ensured. The study group was selected using the purposive sampling method, and the environment and participants were introduced in detail with direct quotes without any comments, while remaining faithful to the nature of the data. In order to ensure the reliability of the study, the creation of the data collection tools, the collection and analysis of the data were carried out by the researcher and three field experts. The interview audio recordings of the study were interpreted by a mathematics teacher and it was seen that the comments were consistent with the comments of the researcher. In addition, the results reached in the study were verified with the data obtained by the field experts and the researcher.

Quantitative reasoning skills encompass the analysis of problems to identify and measure crucial characteristics, along with the application of inductive and deductive reasoning methods. Thematic coding, one of the data analysis methods used in case studies, was used to analyze the data.

In this study, students' symbol sense behaviors (Arcavi, 1994; 2005; Kenney, 2008, Darojaturrofiah, 2017; Rini et al., 2021) at problem-solving phases (Polya, 1945) were combined with algebraic thinking indicators (Usiskin, 1999; Kaput, 1999; NCTM, 2000; Radford, 2000; Kamol, 2005; Kieran, 2004; Kriegler, 2007; Kaput, 2008; Baki, 2008; Schliemann et al., 2006; Stephens et al., 2015) in line by expert opinion and a thematic coding table given in Table 3 below was obtained.

Table 3.

Thematic Coding for Analysis of the Study

Phases	Symbol Sense Behaviors		Algebraic Thinking Skills Indicators
Phase 1: Understanding the Problem	***SSB-A. Friendliness with Symbols*** **1**. Understand the appropriate usage and timing for employing symbols. **2.** Understand the appropriate moment to discontinue the use of symbols. **3.** Identify the significance of symbols within the given problem **4.** Expressing symbols based on their semantic significance within the given mathematical problem ***SSB-B. Designing Symbolic Expressions*** **1.** Associating symbols with problems.		**ATSI-1:** To know the unknown meaning of the variable x. **ATSI-2:** Can use the algebraic rules of the four basic arithmetic operations in the real number system (inverse-element, unit, distributive property of multiplication over addition). **ATSI-3:** Understanding and using the unknown variable x. **ATSI-4:** The meaning of the solution $x = a$ is not interpreted. **ATSI-5:** The meaning of the solution $x = a$ is interpreted. **ATSI-6:** Interpret and use the meaning of the notations. **ATSI-7:** Basic operational skills in the process of solving equations with one unknown **ATSI-8:** Basic procedural skills in solving inequalities with one unknown (writing the fraction form of decimal numbers, knowing that the inequality will change direction by multiplying or dividing both sides of the inequality by a negative number). **ATSI-9:** Demonstrated the ability to determine the symbol representing the unknown quantities in the given problem and to know its meaning. **ATSI-10:** To have the ability to determine the symbol representing the unknown quantities in the given problem in a meaningful way. In addition to this skill, to be able to use the skill of determining the equation by reading the symbol meanings.
Phase 2: Planning Problem Solving	***SSB-C. Symbol Selection*** **1.** Selecting the most suitable symbol for addressing the problem at hand **2.** Opting for the proper depiction of the symbol selected for the problem ***SSB-D. Reading and Using Symbolic Expressions*** **1.** The symbols are expressed within the mathematical models formulated in the given problem **2.** Elucidate the significance of the mathematical model formulated within the context of the problem at hand		

Phase 3: Implementing the Problem-Solving Plan	***SSB-E. Symbol Selection*** **1.** Using mathematical models to solve the problem ***SSB-F. Reading and Using Symbolic Expressions*** **1.** Utilizing the selected approach for problem resolution ***SSB- G. Friendliness with Symbols*** **1.** Applying symbols accurately at every stage of problem-solving ***SSB-H. Designing Symbolic Expressions*** **1.** The capability to effectively construct the verbal and visual data required for the resolution		**ATSI-11:** To have the ability to determine the symbol representing the unknown quantities in the given problem in a meaningful way. In addition to this skill, to be able to use the skills of determining the equation and writing equivalent equations by reading the symbol meanings. **ATSI-12:** Could not show algebraic thinking skills. **ATSI-13:** Considering the definition of consecutive number, three consecutive numbers were expressed symbolically. Using these symbolic expressions, the students demonstrated their ability to construct and solve the related equation. **ATSI-14:** Problem solving in special cases. **ATSI-15:** To create a model by considering the quantitative changes given in the problem and solving the problem by applying the necessary operations.
Phase 4: Going Back and Checking the Solution	***SSB-I. Checking Symbol Meanings*** **1.** Demonstrate the validity of the symbols employed in executing the problem-solving process. ***SSB-J. Symbol Context*** **1.** The interpretation of symbols may vary across different problem scenario		

Note:
1) **"**SSB" means "Symbol Sense Behaviors"
2) "ATSI" means "Algebraic Thinking Skills Indicators"

In alignment with the essence of qualitative inquiry the assessment of the study's trustworthiness and dependability is denoted as 'credibility' in lieu of 'internal validity', 'transferability' in place of 'external validity', 'consistency' rather than 'internal reliability', and 'confirmability' as opposed to 'external reliability' (Guba, 1981; Guba and Lincoln, 1981). In order to ensure the validity of the study, the researcher spent sufficient time in the environment where the study was conducted to collect data and interacted with the study group for a long time. In order to support the accuracy and consistency of the findings, each participant was interviewed twice, and the selection and implementation of the data collection tools and the applications were examined by experts in the field of education. The research's study group was chosen using the purposive sampling technique. The setting and the participants were introduced in detail with direct quotations, without commentary, remaining faithful to the nature of the data. In order to ensure the reliability of the study, the consistency between the researcher and the field experts was taken into consideration in the creation of data collection tools, data collection and analysis stages. In order to determine the comprehensibility of the problems in the research applications by the students and the possible situations/problems that may arise during the implementation of these activities, a pilot application was carried out. As a result of the pilot applications, the experts reached a consensus that it would be more appropriate to examine the symbol sense behaviors in the problems created instead of creating problems for each symbol behavior.

Results

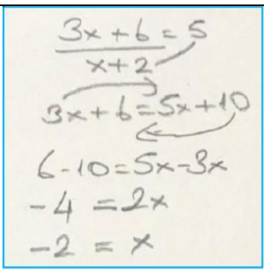
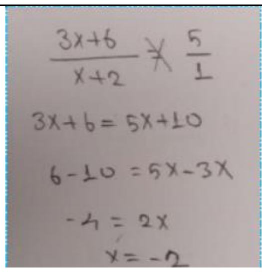
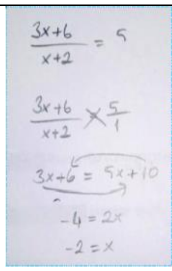
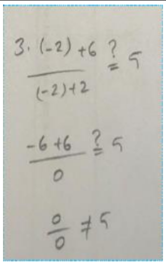
The data of the research obtained from student's solution of five problem. In this segment, the outcomes derived from analyzing the data gathered as part of the study are showcased.

Findings from Problem 1

Students' solutions to Problem 1 are summarized in Table 4.

Table 4.

Problem 1 and Student Solutions

Serkan (LALS)	Eda (MALS)	Yıldız (HALS)
Serkan answered, *"I checked all the steps, the product of ins and outs was done and x was found. Therefore, it is definitely correct".* When Serkan was asked to solve the problem, he first did the inner-outer multiplication, then he used the distributive property of multiplication over addition to reach the result $x = -2$.	Eda replied, *"That's probably right. I checked all the steps, the product of ins and outs was done and* $x = -2$ *was found. However, since* $x = -2$ *makes the denominator zero, the equation is incorrect".* Eda solved the problem by using the distributive property of inner-outer product and multiplication on addition.	Yıldız replied: *"It is never true. If we put the fraction on the left side in brackets of* 3*, the numerator becomes* 3 *times the denominator. In this case, the left side becomes* 3*. Since the right and left sides are not equal, the solution is incorrect".* In the solution process of the problem, he reached the solution $x = -2$ by using the inside-outside product and the distributive property of multiplication on addition. However, he showed that $x = -2$ did not satisfy the equation.
		 

When the student responses presented in Table 4 are analysed, it is noteworthy that students with low and medium academic achievement levels could not see the multiple relationship between the numerator and denominator of the algebraic fraction given in the problem and could not immediately apply the steps in solving an equation in mode. Students with low and medium achievement levels both know the unknown meaning of the variable x and can use the algebraic rules of the four basic arithmetic operations in the real number system (inverse-element, unit, distributive property of multiplication on addition). Serkan (LALS), is not aware that $x = -2$ is not a solution because it makes the value in the denominator zero, so he could not interpret the solution $x = -2$. However, Eda (MALS) interpreted the meaning of the solution $x = -2$. Eda (MALS), was aware of this situation. Yıldız (HALS), stated that the expression $3 = 5$ is a contradiction by considering the multiple relationship between the numerator and denominator of the algebraic fraction given in the problem, and then solved the equation by performing the appropriate operational procedures, reached the result $x = -2$ and showed that this result did not satisfy the equation. She was able to understand and use the variable x as an unknown. Also, like the other two students, she was able to use the algebraic rules of the four basic arithmetic operations in the real number system (inverse-element, unit, distribution property of multiplication over addition). In addition, she was able to interpret and use the parity meaning of the '$=$' notation. When the information is summarized in terms of students' algebraic thinking skills, all three students knew the unknown meaning of the variable x and all students except the student with low academic achievement level were able to manipulate numbers and symbols meaningfully using algebraic rules.

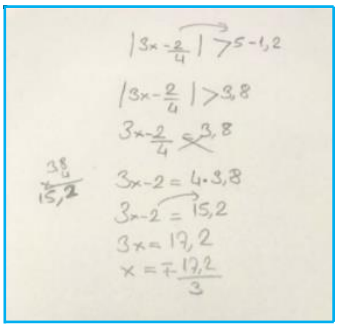
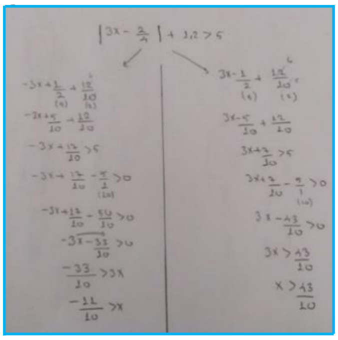
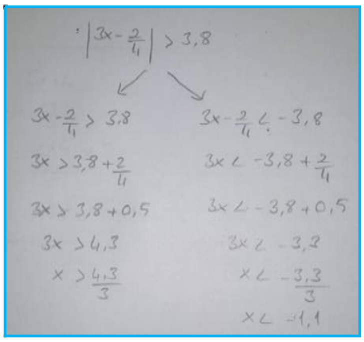
When the study examined in terms of the students' symbol sense behaviors in the problem-solving phases. At the 'understanding the problem' phase, all three students exhibited the behavior of 'friendliness with symbols' by determining that the symbol x means the unknown. At the 'planning to solve the problem' phase, Serkan (LALS) exhibited the behavior of 'reading and using symbolic expressions' by making the expression 5 and $(x + 2)$ correctly by using the distributive property of multiplication on addition'; Eda (MALS) exhibited 'reading and using symbolic expressions' behavior by knowing that the expression given in the question was a rational expression and Yıldız (HALS) is also knowing that the expression given in the question was a rational expression like Eda (MALS). Yıldız (HALS), interpreted the mathematical expression (contradiction) $3 = 5$ by comparing the symbolic expressions $3x + 6$ in the numerator and $x + 2$ in the denominator of the given algebraic fraction. These are indicators of symbol behaviors of 'reading and using symbolic-expressions'. At the 'implementing the problem-solving plan' phase, Serkan (LALS), Eda (MALS) and Yıldız (HALS) both solved the equation established to solve the problem, followed the order of operations and used the properties of operations. The symbol sense behaviors exhibited at this phase for them are 'symbol selection' and 'friendliness with symbols'. At the "going back and checking the solution" phase, Serkan (LALS) did not exhibited any symbol sense. At this phase, Eda (MALS), stated that the result $x = -2$ made the denominator zero and therefore the equation was an incorrect equation after finishing the operation. This means 'checking symbol meanings'. At the "going back and checking the solution" phase, Yıldız (HALS) stated that the numerator and denominator for $x = -2$ were zero, in this case, there was an uncertainty of $0/0$ on the left side of the equation, and the right side of the equation was '5', which proved the correctness of her answer 'It is never correct' when she first read the problem. So at this phase she exhibited the behavior of 'checking the symbol meanings'.

Findings from Problem 2.

Students' solutions for Problem 2 are given in Table 5.

Table 5.

Problem 2 and Student Solutions

Serkan (LALS)	Eda (MALS)	Yıldız (HALS)
Serkan commented on Problem 2: *"There are too many symbols in this problem. It seemed very complicated to me"*. He solved the problem by ignoring inequality and absolute value notations. He made the comment "*When I see the absolute value sign, I remember that the number has one minus and one plus*" and solved the problem as follows.	After reading the second problem, Eda said *"As far as I remember, absolute value has two results.* The *result of this operation is one minus and one plus"* and solved the problem as shown in the figure below.	Yıldız said " *There are absolute value and inequality*. The *question is complicated because there are too many symbols..."* and analysed the problem as follows.
		

When Table 5 is analysed, the student with low academic achievement solved the equation by using the equals symbol instead of the inequality symbol and determined the result as only one value.

It is seen that the students with medium and high academic achievement did not analyse the sign inside the absolute value, solved two separate inequalities depending on the variable and did not compare the solutions obtained from these two inequalities with the solutions obtained from the sign analysis. Therefore, the results obtained from the solutions of two separate inequalities were left as they were and the solution set was not shown. Serkan (LALS) has basic operational skills in the process of solving equations with one unknown. Eda (MALS) and Yıldız (HALS) have basic procedural skills in solving inequalities with one unknown (writing the fraction form of decimal numbers, knowing that the inequality will change direction by multiplying or dividing both sides of the inequality by a negative number). When the information in Table 5 is summarized in terms of students' algebraic thinking skills, all students interpreted the symbol x as an unknown. It is seen that the student with low academic achievement level cannot comprehend the relational difference between equality and inequality and has procedural skills. On the other hand, students with medium and high academic achievement levels can be said to have only basic procedural skills due to their inability to understand the concepts of absolute value and inequality.

The study was also examined in terms of students' symbol perception behaviors in the problem-solving stages. At the ‘understanding the problem’ phase, Eda (MALS) and Yıldız (HALS) exhibited ‘friendliness with symbols’ behavior since they knew that the symbol ‘x’ represented the unknown. Serkan (LALS) displayed the same symbol detection behavior as them. He subtracted 1,2 from 5 to leave 'x' alone. Serkan (LALS) did not exhibit any symbol sense behavior for this problem in the other phases except for the ‘understanding the problem’ phase. Eda (MALS) and Yıldız (HALS) exhibited the behavior of ‘reading and using symbolic expressions’ in the ‘planning problem solving’ phase, as they continued the process by using the properties of the absolute value expression, Eda (MALS) and Yıldız (HALS) exhibited the ‘symbol selection’ behavior at the ‘implementing the problem-solving plan’ phase because they solved the equation established for the solution of the problem, followed the sequence of operations and used the properties of the operations.Unlike Serkan (LALS) and Eda (MALS), Yıldız provided the solution by giving special values to 'x' at the ‘going back and checking the solution’ phase. This shows that Yıldız exhibited the behavior of 'checking symbol meanings' at this phase.

Findings from Problem 3.

Students' solutions for Problem 3 are given in Table 6.

Table 6.

Problem 3 and Student Solutions

Serkan (LALS)	Eda (MALS)	Yıldız (HALS)
After reading Problem 3, Serkan *answered as follows: "If we call the professor P, then the student is $6S$ and therefore I think the correct choice is* "$6S = P$".	After reading the third problem, Eda answered *"If we call the professor P, then the correct option is "$6P = S$".*	After reading the third problem, Yıldız determined the symbols P for the number of professors and S for the number of students and reached the equation $6P = S$. While reviewing the options, she commented that *"if I divide both sides by six, then the option $S/6 = P$ is also suitable for this problem"*. Yıldız marked equations 3 and 5 in the problem.

However, in Table 6, Yıldız (HALS) and Eda (MALS) were able to correctly read and interpret the quantitative relationship between the number of teachers and the number of students given in the problem. Serkan (LALS), on the other hand, could not read the relationship between the quantities given in the problem correctly and could not find the correct equation. In addition, Serkan (LALS), could only determine the symbol representing the unknown quantities in the problem. The students with medium and high academic achievement were able to identify the symbols representing the unknown quantities (number of professors and students), see the relationship between these quantities and write the related equation. However, only Yıldız (HALS) was able to see the equivalent representation of the equation.

The study was also examined in terms of students' symbol perception behaviors in the problem-solving phases. Serkan (LALS) exhibited 'friendliness with symbols' behavior by selecting 'S' and 'P' symbols at the 'understanding the problem' phase. In this sense, Serkan (LALS) showed the ability to know how and when to use symbols. The student named Eda (MALS), on the other hand, exhibited 'friendliness with symbols' behavior at the 'understanding the problem' phase because she used the symbols 'S' and 'P', knew that the symbols 'S' and 'P' were changing quantities, and was able to write that the number of students was 6 times the number of professors. In addition, at this phase, she exhibited the behavior of 'designing symbolic expressions' because she formed the algebraic expression $'6\mathrm{P} = \mathrm{S}'$. At the 'understanding the problem' phase, Yıldız (HALS) formed the algebraic expressions $6\mathrm{P} = \mathrm{S}$ and $\mathrm{S}/6 = \mathrm{P}$ by using the symbols 'S' and 'P'. Yıldız also knew that the symbols 'S' and 'P' are changing quantities and that the number of students is 6 times the number of professors or that one sixth of the number of students is a professor. This shows that she exhibited 'friendliness with symbols' behavior at this phase of problem solving. At the ‘planning problem solving’ phase, Eda (MALS), exhibited the behavior of ‘symbol selection’ because she could choose the symbols ‘S’ and ‘P’, and the behavior of ‘reading and using symbolic expressions’ because she explained the meaning of the mathematical-model he created by expressing the symbols in the equation ‘$6P = S$’. Choosing the symbols 'S' and 'P' and explaining the meaning of the mathematical model Yıldız (HALS), created by expressing these symbols in the equations $6P = S$ and $S/6 = P$ shows that she exhibited the behaviors of 'symbol selection' and 'reading and using symbolic expressions' at the 'planning problem solving' phase.

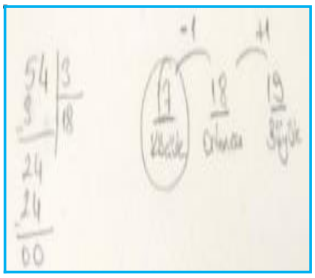
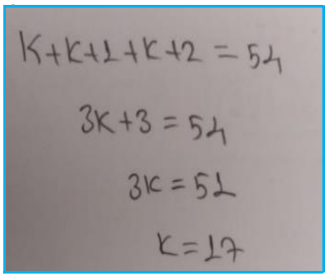
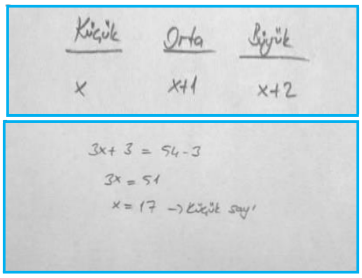
At the phase of ‘implementing the problem-solving plan’, Eda (MALS), exhibited the behavior of ‘symbol selection’ because she multiplied the number of professors by 6. Finding the number of students by multiplying the number of professors by 6 and finding the number of professors by dividing the number of students by 6 shows that Yıldız (HALS), exhibited the behavior of ‘symbol selection’ in the ‘implementing the problem-solving plan’ phase. In the ‘going back and checking the solution’ phase, when asked whether Eda (MALS) would use the un-knowns ‘P’ and ‘S’ in another problem, she replied ‘...if there is an equation starting with the letters P and S. For example, if there was a problem such as the amount of cake per student as a result of distributing 1 cake to 6 students, it would be as if’. This shows that Eda (MALS), exhibited 'symbol context' behavior at this phase. As a result, Eda (MALS) was able to identify the symbol representing the unknown quantities in the given problem in a meaningful way. In addition to this skill, she was also able to determine the equation by reading the symbol meanings. The fact that he rechecked the correctness of the options Yıldız (HALS), marked and the algebraic expressions she found and stated that she could use the un-knowns 'P' and 'S' for another problem shows that she exhibited the behaviors of 'checking symbol meanings' and 'symbol context' in the 'going back and checking the solution' phase. In this context, Yıldız (HALS), has the ability to determine the symbol representing the unknown quantities in the given problem in a meaningful way. In addition to this skill, she was able to use the skills of determining equations and writing equations by reading symbol meanings.

Findings from Problem 4.

Students' solutions for Problem 4 are given in Table 7.

Table 7.

Problem 4 and Student Solutions

Serkan (LALS)	Eda (MALS)	Yıldız (HALS)
Serkan, read the fourth problem and said *"There is a rule used in solving such problems. I divide* 54 *by* 3 *and find the median number, then I decrease by 1 to reach the small number"* and solved the problem. Serkan's solution is given below.	After reading the fourth problem, Eda *gave the values of "k", "$k+1$" and "$k+2$"* in order to solve the problem *based on the definition that consecutive numbers, that is, numbers increasing one by one, are consecutive numbers and solved the problem. The solution is given in the figure below.*	Yıldız read the fourth problem and said *"It says consecutive number. Numbers that come one after the other and have a difference between them are called consecutive numbers."* Yıldız identified three consecutive numbers in the figure to solve *the problem.* The solution of the problem is given below.
		

In Table 7, academically successful and moderately successful students demonstrated the ability to symbolise three consecutive numbers, write and solve the related equation. However, since it was not clear how the student with low academic achievement used algebraic reasoning in the solution, the researcher interviewed the student about the solution of the problem. The dialogue between the researcher and Serkan is given below.

Researcher: Why did you prefer such a solution?

Serkan (LALS): It is a solution I remember from secondary school. Given the sum of three consecutive numbers, if I divide the sum by three, the median number is reached. It gives me an advantage in solving such problems.

Researcher: Well...How can you solve this question using symbols?

Serkan (LALS): If the sum of three numbers is 54...It says the sum of three numbers. If I use three letters instead of numbers...

After these comments, Serkan wrote the equation given in Figure 1.

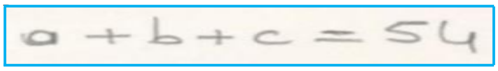


Figure 1. Serkan's equation for Problem 4.

Serkan (LALS), could not continue the process after writing the equation and said "*I always get confused when letters are involved. Therefore, I cannot solve this question in this way*".

The interview between Eda (MALS), a student with moderate academic achievement, and the researcher about the use of variables in the sense of unknown is as follows:

Researcher: Why did you use the variables "k", "$k + 1$", "$k + 2$" in solving the problem? Can you explain?

Eda (MALS): Normally, I use the letters "x" or "y" when I make equations, but when I read the word consecutive, I think of increasing or decreasing numbers depending on "k" or "n".

Researcher: Would you use "k" as an unknown in another problem?

Eda (MALS): I use "k" especially in problems about consecutive numbers or numbers that are multiples of each other.

The interview between Yıldız (HALS), a student with high academic achievement, and the researcher about the use of variables in the sense of unknown is as follows.

Researcher: Can you explain why you indicated the unknown with "x" in the solution of the problem?

Yıldız (HALS): When I think of unknown, I always think of "x".

Researcher: Would you represent the unknown with "x" in another problem solution?

Yıldız (HALS): Yes.

In this interview between the researcher and Yıldız (HALS), the view that the symbol "x" is common in the use of variables in the sense of unknown was revealed.

Researcher: If the largest number was asked in this problem, how would you determine the consecutive numbers?

Yıldız (HALS): Since the unknown would be the largest number, I would determine the largest number as "x", the median number as "$x - 1$" and the smallest number as "$x - 2$".

In the light of the interviews between the researcher and the students, it is seen that students with medium and high academic achievement levels comprehended the concept of consecutive number as "the number that follows a number and has 1 more". Accordingly, it is seen that the students have the behavior of determining the appropriate representation form of the selected symbol. On the other hand, since the student with low academic achievement level did not have the concept of consecutive number, he/she did not have the ability to represent the variable in accordance with the problem context. Serkan (LALS) could not show algebraic thinking skills. Eda (MALS) and Yıldız (HALS) considered the definition of consecutive number, three consecutive numbers were expressed symbolically. And they used these symbolic expressions, the students demonstrated their ability to construct and solve the related equation. When the study is summarized in terms of students' algebraic thinking skills, students were able to use the concept of consecutive number in a meaningful way. The symbolic expression of the problem sentence was reflected with the related equation. The equation was analysed by applying algebraic and arithmetical operations.

When the study was analysed in terms of students' symbol perception behaviors in problem solving phases; at the 'understanding the problem' phase, Serkan (LALS) exhibited 'friendliness with symbols' and 'designing symbolic expressions' behavior by using the algebraic expression '$a + b + c$' for the sum of three numbers; Eda (MALS), said: "*Numbers that come one after the other, that is, numbers that increase one by one, are called consecutive numbers. Therefore, if I call the small number 'k', the other numbers will be '$k + 1$' and '$k + 2$'* and displayed the behaviors of "friendliness with symbols" and "designing symbolic expressions"; Yıldız (HALS) was able to determine the expressions 'x, $x + 1$ and $x + 2$' for three consecutive numbers, so she exhibited the behaviors of 'friendliness with symbols' and 'designing symbolic expressions' too. At the 'planning problem solving' phase, Serkan (LALS) used the letters a, b, c for three numbers and wrote the equation '$a + b + c = 54$' so he exhibited the behavior of 'reading and using symbolic expressions'; Eda (MALS) exhibited the behavior of 'symbol selection' by choosing the right symbol to solve the problem, choosing the appropriate representation method of the symbol chosen in the problem because she called the small number as "k", and the other numbers as "$k + 1$", "$k + 2$". Yıldız (HALS) exhibited the behaviors of choosing the correct symbol to solve the problem, expressing symbols in the mathematical models created in the problem, explaining the meaning of the mathematical model created in the problem and choosing the appropriate representation method of the symbol chosen in the problem. These indicators show that Yıldız exhibited the behaviors of 'symbol selection' and 'reading and using symbolic expressions' in the 'planning problem solving' phase. In the "implementing the problem-solving plan" phase, Serkan (LALS) could not exhibited any symbol sense at this phase. Eda (MALS), formed the equation by choosing the correct symbols, performed the necessary operations to solve the problem, and used the order of operations and the properties of the operations correctly while solving the equation established to solve the problem. This shows that the student exhibited 'symbol selection', 'reading and using symbolic expressions' and 'friendliness with symbols' behaviors at this phase of problem solving. Yıldız (HALS), used the selected method to solve the problem, constructed and solved the equation correctly by equaling the sum of the three consecutive numbers she has chosen to 54 and solves the equation correctly. This corresponds, to the behaviors of 'symbol selection', 'reading and using symbolic expressions' and 'friendliness with symbols' at the phase of “implementing the problem-solving plan”. Serkan (LALS), could not exhibit any behavior at this phase. In the “going back and checking the solution” phase, Eda (MALS), demonstrated 'symbol context' behavior by stating that she could use ‘k’ for another problem. Yıldız (HALS), solved the equation and found the small number and then verified the operation and said that she could use ‘x’ for any problem shows that she exhibited the behaviors of ‘checking the symbol meanings’ and ‘symbol context’ in the “going back and checking the solution” phase.

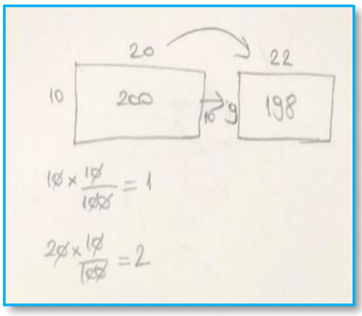
Findings from Problem 5.

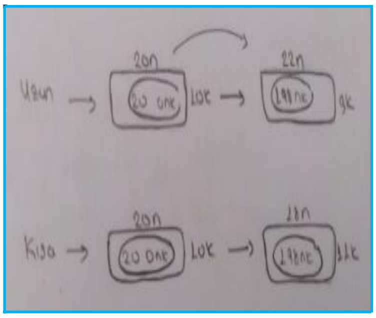
Students' solutions for Problem 5, are given in Table 8.

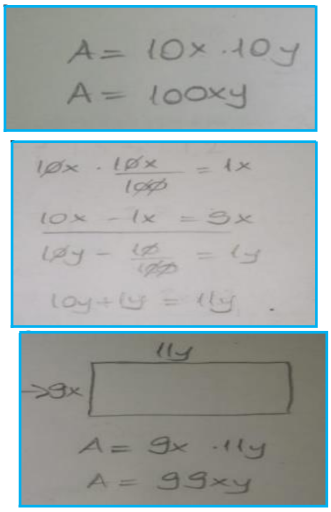
Table 8.

Problem 5 and Student Solutions

Serkan (LALS)	Eda (MALS)	Yıldız (HALS)
When Serkan first read the fifth problem, he stated that there could not be a change in the area value since the amount of increase and decrease in the side lengths of the rectangle was the same (10%). Then, for ease of operation, he drew a rectangle with side lengths of 10 br and 20 br and concluded that there would be a decrease in the area by numerical calculations. The solution is given below.	When Eda first read the problem, she stated that there would be no change in the area value of the rectangle since there would be a 10% increase and decrease in the side lengths of the rectangle. She chose a rectangle with side lengths of 10n and 10k in order to easily calculate the 10% increase and decrease. "*Now I am confused here because it says one side or something. I don't know which side it is, so I draw two different rectangles because it says one side*." As can be seen from the solution given below, he was able to see the decrease in the area value after making numerical calculations for two different situations.	Yıldız stated that he would not answer the problem without making numerical calculations. He drew a rectangle with 10x and 10y side lengths, increased one side length by 10% and decreased the other side length by 10%, determined the change in the area value of the rectangle numerically and answered the question. The solution is given below.







When Table 8 is analysed, it is seen that the students with low and medium academic achievement made a comment that there would be no change in the area value of the rectangle due to the same amount (10%) increase and decrease in the side lengths of the rectangle. The student with high academic achievement stated that he could not make a comment without making any calculations.

When the study is summarized in terms of students' algebraic thinking skills, Serkan (LALS), did problem-solving in special cases. Eda (MALS) and Yıldız (HALS) created a model by considering the quantitative changes given in the problem and solving the problem by applying the necessary operations. They have the ability to create models by considering the quantitative changes given in the problem, and to determine the quantitative relationships expressed verbally, symbolically and numerically.

Symbol sense behaviors of the students observed in Table 8 according to their academic achievement levels: a) The fact that Serkan (LALS) solved over a rectangle with lengths of 10 units and 20 units is an indication of his symbol abandonment behavior. In order to easily calculate the 10% increase and decrease in the side lengths of the rectangle, Eda (MALS), drew a rectangle with side lengths 10n and 20k, while the student with high academic achievement drew a rectangle with side lengths 10x and 10y. This is an indication that students with medium and high academic achievement have the behaviors of "knowing how and when to use symbols", "associating symbols with the problem" and "writing symbols according to the meaning in the problem" as symbol sense.

The study was also analysed in terms of students' symbol perception behaviors in problem solving phases. At the 'understanding the problem' phase, Serkan (LALS), drew a rectangle to solve the problem and abandoned the symbols. The fact that he stated that it was easy to get the percentages of the numbers 10 and 20 that he chose for the solution of the problem points to the behavior of "designing symbolic expressions" at the phase of 'understanding the problem'. Eda (MALS), drew two different rectangles and used the values '10k' and '20n' for the short and long sides of the rectangle, respectively. When asked why she used the values '10k' and '20n', she stated that it was easier to calculate the percentage with these values. This shows that the student exhibited the behaviors of 'friendliness with symbols' and 'designing symbolic expressions' at the "understanding the problem" phase. Yıldız (HALS) used the values '10x' and '10y' to solve the problem, drew a rectangle to solve the problem, and determined the short side of the rectangle as '10x' and the long side as '10y'. She said that she determined these expressions because it was easy to get the percentages of '10x' and '10y'. These indicators correspond to the behaviors of 'friendliness with symbols' and 'designing symbolic expressions' in the 'understanding the problem' phase. In the 'planning to solve the problem' phase, Serkan (LALS), stated that it was easy to get the percentage of the numbers '10' and '20' he chose to solve the problem, and he also drew a rectangle and gave numerical values as '10' and '20' on its sides. Thus, Serkan (LALS) exhibited the behavior of 'reading and using symbolic expressions' in this phase. Eda (MALS), chose the correct symbols to solve the problem and expressed the symbols in the mathematical models created in the problem. These approaches show that she exhibited 'symbol selection' and 'reading symbolic expressions' behaviors at this phase. Yıldız (HALS), said that *'I should use numbers whose products are 100 to do operations with percentages because it makes the operation easier and determined the values $10x$ and $10y$ for the side lengths of the rectangle'*. Then she wrote that the expression $10x.10y = 100xy$ is the area of a rectangle with a short side length of 10x and a long side length of 10y. These correspond to the behaviors of 'symbol selection' and 'reading and using symbolic expressions' in the 'planning problem solving' phase. At the "implementing the problem-solving plan" phase; Eda (MALS) and Yıldız (HALS) calculated the percentages of increase and decrease in the side lengths of the rectangles, finding the areas of the rectangles correctly after the operations and drawing two new rectangles with the same area after the operations show that the student exhibited the behaviors of 'symbol selection', 'reading and using symbolic expressions', 'friendliness with symbols' and 'designing symbolic expressions' at this phase. Serkan (LALS) could not exhibit any symbol sense at the "applying the problem-solving plan" and "going back and checking the solution" phases. Eda (MALS) exhibited 'symbol context' behavior at the "going back and checking the solution" phase by saying that the side lengths of the rectangle could also be '10x' and '20y'. When Yıldız (HALS) stated that the area decreased after finishing the process and when asked whether he could give another value for the side lengths of the rectangle, he said that he could also use the values '10m' and '10n', which corresponds to the 'symbol context' behavior in relation to explaining that the symbols used will have different meanings in different problems.

The fact that the students correctly calculated the change in the side lengths of the rectangles they identified as models and saw the decrease in the area value is an indication of their behaviors of "using mathematical models to solve the problem" and "expressing the symbols in the mathematical models created in the problem".

Thematic coding, a data analysis technique commonly employed in case studies, was utilized for the examination of the data. Thematic analysis constitutes a form of qualitative analysis that focuses on the identification of patterns within the data and the development of themes. According to Boyatzis (1998), thematic coding is characterized not as a unique methodology but rather as a versatile instrument that can be applied across various research approaches. Findings from five problems of this study, Table 9 shows that the summarize of them. Analysis of the summarize is made with thematic coding from Table 3.

Table 9

Summarising the Findings Obtained from the Problems by Thematic Coding

Students	**Serkan (LLAS)**		**Eda (MLAS)**		**Yıldız (HLAS)**	
Categories	**SSB**	**ATSI**	**SSB**	**ATSI**	**SSB**	**ATSI**
Problem 1	SSB-A.3 SSB-D.1 SSB-E.1 SSB-G.1	ATSI-1 ATSI-2 ATSI-4	SSB-A.3 SSB-D.1 SSB-E.1 SSB-G.1 SSB-I.1	ATSI-1 ATSI-2 ATSI-5	SSB-A.3 SSB-D.1 SSB-E.1 SSB-G.1 SSB-I.1	ATSI-2 ATSI-3 ATSI-6
Problem 2	SSB-A.3	ATSI-7	SSB-A.3 SSB-D.1 SSB-E.1	ATSI-8	SSB-A.3 SSB-D.1 SSB-E.1 SSB-I.1	ATSI-8
Problem 3	SSB-A.1	ATSI-9	SSB-A.1 SSB-A.3 SSB-A.4 SSB-B.1 SSB-C.1	ATSI-10	SSB-A.1 SSB-B.1 SSB-A.3 SSB-A.4 SSB-C.1	ATSI-11

			SSB-D.1 SSB-D.2 SSB-E.1 SSB-J.1		SSB-D.1 SSB-D.2 SSB-E.1 SSB-I.1 SSB-J.1	
Problem 4	SSB-A.1 SSB-B.1 SSB-D.1 SSB-D.2	ATSI-12	SSB-A.1 SSB-A.3 SSB-B.1 SSB-C.1 SSB-C.2 SSB-E.1 SSB-F.1 SSB-G.1 SSB-J.1	ATSI-13	SSB-A.1 SSB-A.3 SSB-B.1 SSB-C.1 SSB-C.2 SSB-D.1 SSB-D.2 SSB-E.1 SSB-F.1 SSB-G.1 SSB-I.1 SSB-J.1	ATSI-13
Problem 5	SSB-A.2 SSB-B.1 SSB-D.1	ATSI-14	SSB-A.1 SSB-A.2 SSB-A.3 SSB-B.1 SSB-C.1 SSB-D.1 SSB-E.1 SSB-F.1 SSB-G.1 SSB-H.1 SSB-J.1	ATSI-15	SSB-A.1 SSB-A.2 SSB-A.3 SSB-A.4 SSB-C.1 SSB-D.1 SSB-D.2 SSB-E.1 SSB-F.1 SSB-G.1 SSB-H.1 SSB-I.1 SSB-J.1	ATSI-15

Discussion and Conclusion

In the first step of this study, which was carried out with three ninth grade students with low, medium and high achievement levels, pre-interview questions were asked to the students. According to the findings obtained from the pre-interview questions, the student with low achievement level disliked mathematics, mostly because he could not concretize the symbols, and felt more successful in class levels where arithmetic was at the forefront; the student with medium achievement level saw mathematics as a set of formulas, could not fully comprehend the symbols, and could sometimes use algebra while solving problems; On the other hand, students with high achievement level were always interested in mathematics, liked the subject of solving equations, which is the focus of algebra teaching, and were able to make connections between algebraic properties and rules, symbolic and numerical representations.

In second step of the study, in which algebraic thinking skills and symbol sense behaviors of ninth grade students were investigated from the perspective of academic achievement, results were obtained that are thought to contribute to the mathematics teaching literature in terms of how students use and interpret symbols in the algebraic thinking process.

I. The Relationship between Academic Achievement and Algebraic Thinking Skills

In the solution of the equation with algebraic fraction given in the first problem, only the student with high academic achievement level showed meaningful symbolic reading and interpretation skills by establishing the relationship between the symbolic expressions in the numerator and denominator and solved the problem without performing standard procedural steps. In other words, the academically successful student showed the ability to use the symbolic expressions in the given algebraic equation flexibly. When the student was asked to solve the given equation, he/she solved the equation by using the relevant operation properties correctly. Arithmetic is necessary to understand the basic relationships of numbers, while algebra is necessary to develop more complex mathematical thinking skills. The combination of these two areas strengthens students' mathematical problem-solving skills and prepares them for more advanced mathematical topics. Being good at both arithmetic and algebraic terms require having basic arithmetic knowledge, algebraic thinking skills, problem-solving skills, logic and critical thinking, and abstract thinking skills, as well as being successful in application and practice and error analysis. Based on the observation that this student can also perform basic operations quickly and accurately; is careful when solving equations, simplifying expressions, and performing mathematical operations; understands mathematical problems and develops appropriate solutions to the problem; evaluates results logically by thinking step by step; can grasp mathematical structures and relationships; and knows how to use arithmetic and algebraic operations in daily life and in different contexts, it can be said that this student's basic operation skills are good in both arithmetic and algebraic terms. Students with low and medium achievement levels analysed the equation using standard algebraic properties without showing symbolic reading and interpretation skills. However, while the student with an average academic achievement level provided the result obtained in the solution of the equation, the student with a low academic achievement level did not provide the solution. The student with low academic achievement did not interpret the solution while performing the algebraic manipulations correctly. This behavior caused him to choose the wrong option in solving the problem. In the solution process of this problem, the students correctly solved the algebraic equation manipulatively by using the related operations and algorithms correctly. However, in this process, while the student with high academic achievement solved the algebraic equation by using the symbols flexibly and fluently, the student with low academic achievement solved the mode in a meaningless way.

In the second problem of the study, students were given an inequality (>) with absolute value and asked the solution set. The student with a low level of academic achievement analysed the inequality with absolute value as a linear equation, ignoring the concept of absolute value. On the other hand, students with medium and high academic achievement level transformed the absolute value inequality into two separate expressions without analysing the sign inside the absolute value and made certain algebraic calculations without establishing any logical relationship between these two inequalities. However, they could not determine the solution set. These behaviors are consistent with the studies of Şandır, Ubuz and Argün (2002), Demetgül and Baki (2020). From this point of view, it was concluded that ninth grade students tended to use the rules learned in the concept of absolute value and inequality in a meaningless way rather than meaningful analysis in both the definition and properties of absolute value and inequality solutions.

In the third problem of the study, also known as the "*student-professor*" problem in the literature, students were expected to be able to recognise the quantitative relationship in the algebraic expression of the problem. In the solution of this problem, the student with a low level of achievement fell into a linguistic trap and marked the wrong option by making a reversal error. While the student with a medium achievement level was able to recognise only one of the correct options through meaningful algebraic reading and interpretation, the student with a high level of academic achievement marked both correct options. This result is consistent with Rosnick and Clement (1980); Clement, Lochhead and Monk (1981) studies.

In the fourth problem of the study, the algebraic thinking skill expected from the students is the symbolic expression of three consecutive numbers in generalised arithmetic and the solution of the related equation in the context of the problem. The student with low academic achievement level could not express three consecutive numbers symbolically and could not write the related equation. The student made a meaningless solution completely by rote memorisation depending on the rule given in the lesson. Christou and Vosniadou (2005) posit that students demonstrate a tendency to construe real symbols within algebra solely in the context of natural numbers, a phenomenon heavily shaped by the structural characteristics of the algebraic entity. This phenomenon exerts a notable impact on the comprehension of symbols and the thought processes involved in algebraic manipulation.

On the other hand, students with medium and high academic achievement were able to express three consecutive numbers symbolically as generalised numbers by using the concept of consecutive number in a meaningful way and made the necessary manipulative analyses by writing the related equation.

In the fifth problem of the study, the algebraic thinking skill expected from the students is to be able to reason algebraically about the quantitative relations in the verbal expression of the problem. To be able to construct and solve the relevant symbolic equation by correctly analysing the quantitative relations given in the problem. The student with low academic achievement analysed the problem arithmetically by determining numerical values appropriate to the problem context. The students with medium and high academic achievement expressed the quantitative reasoning in the problem sentence with appropriate symbolic representations and solved the problem by writing the relevant equation.

In the light of the detailed analyses of the five study questions given above, it was concluded that the algebraic thinking skill that the student with low achievement level had the most difficulty was the ability to use symbols and algebraic relationships. In addition to this result, it was determined that the algebraic thinking skills of the students with high achievement level were higher than the other students. These results are consistent with the study of Bağdat and Anapa-Saban (2014). Another result obtained in the study regarding algebraic thinking skills is that students with high and medium academic achievement levels did not have difficulty in the process of solving algebraic equations, while medium and low achievement students had difficulty in solving inequalities with absolute values. This result is consistent with Kenney (2008).

II. The Relationship between Academic Achievement and Symbol Sense Behaviors

When the results of the study on symbol sense behaviors from the perspective of academic achievement were examined, it was found that the student with low achievement level could not use symbols and letters in problem solving phases and preferred arithmetic solutions instead of algebraic solutions. It was concluded that the student could not present the information extracted from the problems in a mathematically correct way and, accordingly, could not think algebraically in depth in applying and interpreting mathematical findings. The absence of symbol sense results in the adoption of ad hoc approaches rather than systematic algebraic methods (Turşucu, Spandaw & de Vires, 2018). The students demonstrated a diminished symbol sense attributed to insufficient conceptual understanding. These results are similar to, Naidoo (2009); Darojaturrofiah's (2017) and Sugilar, et al., (2019) studies. Kenney (2008) stated that students with low achievement level can analyse and formulate quantitative relationships in problems appropriately, but they have difficulty in drawing logical conclusions in the solving and interpretation phases. In this context, the results of the study are also compatible with the results of Kenney (2008).

The student possessing an medium level of academic achievement successfully reformulated the quantitative relationships in each problem using symbolic and verbal representations. While resolving the problem, the student managed to analyze the information in the problem's new expression, identify, and implement the required solution strategies. The outcomes derived from this student with a moderate level of academic achievement align with the findings of Darojaturrofiah (2017). In addition, Eda (MALS), successfully completed the step of going back and checking the solution in most of the questions. Rini, et al., (2021) stated that students who successfully complete the step of going back and checking the solution from problem solving steps generally show symbol sense behaviors. In this sense, Eda (MALS), was found to have more competence in symbol sense behaviors than Serkan (LALS).

The student who demonstrated a high degree of academic accomplishment exhibited a proficient and flexible utilization of symbols and letters throughout the problem-solving process. Moreover, she displayed the ability to systematically analyze the quantitative relationships within each problem, as well as to formulate and resolve the requisite equations. Consequently, meaningful and logical outcomes were derived for each problem. Within this framework, the student with elevated academic attainment displayed accurate symbol interpretation behaviors while addressing the problems. This research finding aligns with the works of Kenney (2008), Darojaturrofiah (2017), Sugilar, et al., (2019) and Rini et al., (2021).

The findings derived from the research demonstrate that individuals exhibiting a high level of academic accomplishment possess enhanced skills in algebraic reasoning and symbol interpretation.

In this particular context, a direct correlation exists between the level of academic achievement and the development of algebraic thinking skills and behaviors related to symbol sense.

In the third and last step of the study, three students were asked the final interview questions. The findings obtained from the results of the interview were used to obtain more information about the symbol sense of the students while solving algebraic problems. According to the findings obtained from the final interview questions, it was concluded that the student with a low level of achievement saw the symbols as letters and was more successful in the problem requiring numerical calculations without letters, the student with a medium level of achievement saw the symbols as signs and the problem of consecutive numbers, which she had more experience before, was easier for her, and the student with a high level of achievement saw the symbols as both letters, operation symbols and other symbols, and the problem in which she could make modelling was easier for her.

Recommendations

This section presents recommendations for practitioners and researchers based on the findings and results of the study.

Recommendations for practitioners: To develop a symbol sense in teachers, teacher training should include information on the importance of symbolism, its components, and how to impart it to students. New tasks and problems that will enhance symbol sense can be created by teachers using the services of technology.

Recommendations for researchers: In future studies, starting from the sixth grade, which is the first stage of algebra education, the symbol sense behaviors of students at different levels (middle school, high school, undergraduate), especially the 6^{th}, 7^{th} and 8^{th} grade levels, can be examined. In future studies, tasks from different algebraic topics such as logarithms, polynomials, functions, logic, linear algebra, derivatives, integrals, polynomials, trigonometry, derivatives, integrals, complex numbers, etc. can be prepared to examine students' symbolic sense behavior. Future studies may examine the relationships between "number sense", "symbol sense", and "structure sense" together and look at algebraic thinking from a different perspective.

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